

# EE 330

## Lecture 37

### Digital Circuit Design

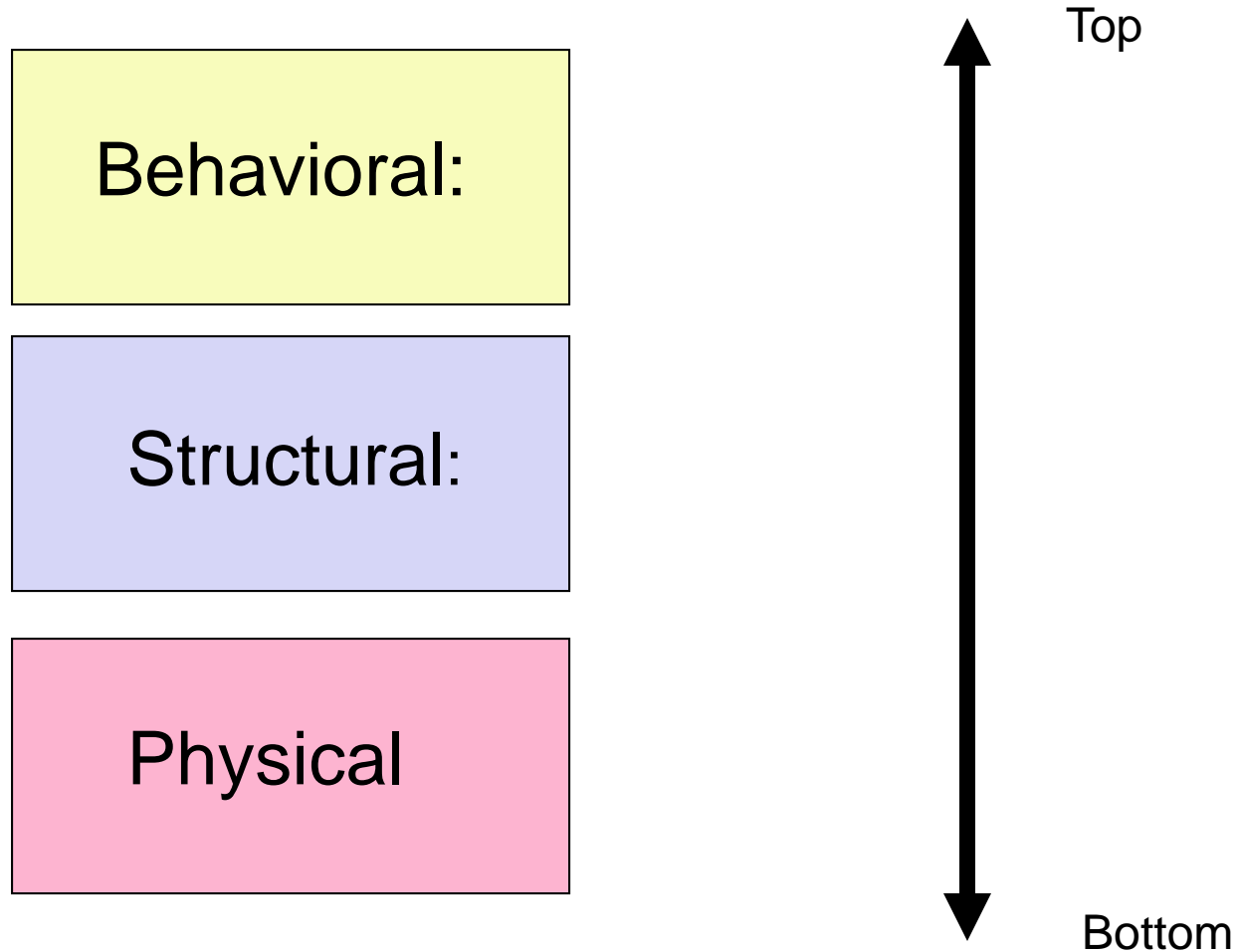
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter

# Fall 2023 Exam Schedule

Exam 1	Friday Sept 22	
Exam 2	Friday Oct 20	
Exam 3	Friday Nov. 17	
Final	Monday Dec 11	12:00 – 2:00 p.m.

Review from Last Lecture

# Hierarchical Digital Design Domains:



Multiple Levels of Abstraction

# Hierarchical Digital Design Domains:

**Behavioral** : Describes what a system does or what it should do

**Structural** : Identifies constituent blocks and describes how these blocks are interconnected and how they interact

**Physical** : Describes the constituent blocks to both the transistor and polygon level and their physical placement and interconnection

Multiple representations often exist at any level or sublevel

# Logic Optimization

What is optimized (or minimized) ?





- Number of Gates
- Number or Levels of Logic
- Speed
- Delay
- Power Dissipation
- Area
- Cost
- Peak Current
- • •

Depends Upon What User Is Interested In

# Standard Cell Library

- Set of primitive building blocks that have been pre-characterized for dc and high frequency performance
- Generally includes basic multiple-input gates and flip flops
- P-cells often included
- Can include higher-level blocks
  - Adders, multipliers, shift registers, counters, ...
- Cell library often augmented by specific needs of a group or customer

# Digital Circuit Design

-  • Hierarchical Design
  -  • Basic Logic Gates
  -  • Properties of Logic Families
  -  • Characterization of CMOS Inverter
    - Static CMOS Logic Gates
      - Ratio Logic
    - Propagation Delay
      - Simple analytical models
        - FI/OD
        - Logical Effort
      - Elmore Delay
    - Sizing of Gates
      - The Reference Inverter
  - Propagation Delay with Multiple Levels of Logic
  - Optimal driving of Large Capacitive Loads
  - Power Dissipation in Logic Circuits
  - Other Logic Styles
  - Array Logic
  - Ring Oscillators
- 

 done

 partial

# Logic Circuit Block Design

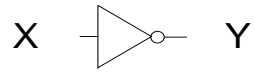
Many different logic design styles

- Static Logic Gates
- Complex Logic Gates
- Pseudo NMOS
- Pass Transistor Logic
- Dynamic Logic Gates
  - Domino Logic
  - Zipper Logic
  - Output Prediction Logic

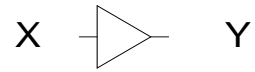
Various logic design styles often combined in the implementation of one logic block



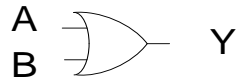
# The basic logic gates



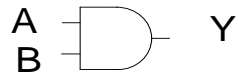
$$Y = \overline{X}$$



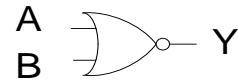
$$Y = X$$



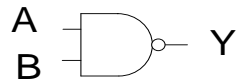
$$Y = A + B$$



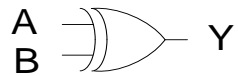
$$Y = A \cdot B$$



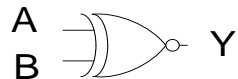
$$Y = \overline{A + B}$$



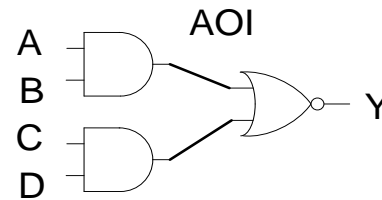
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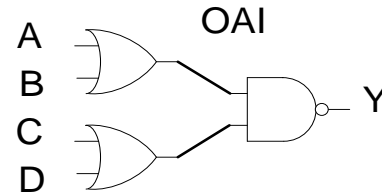
$$Y = A \oplus B$$



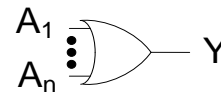
$$Y = \overline{A \oplus B}$$



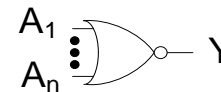
$$Y = \overline{A \cdot B + C \cdot D}$$



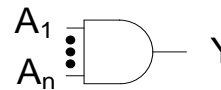
$$Y = \overline{(A + B) \cdot (C + D)}$$



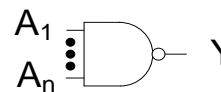
$$Y = A_1 + A_2 + \dots + A_n$$



$$Y = \overline{A_1 + A_2 + \dots + A_n}$$

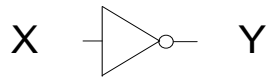


$$Y = A_1 \cdot A_2 \cdot \dots \cdot A_n$$

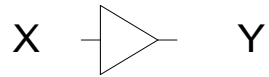


$$Y = \overline{A_1 \cdot A_2 \cdot \dots \cdot A_n}$$

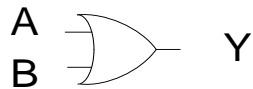
# The basic logic gates



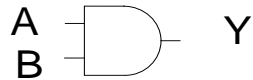
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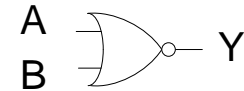
$$Y = X$$



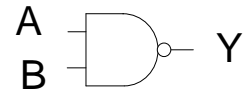
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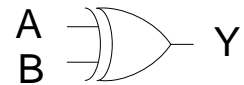
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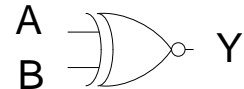
$$Y = \overline{A + B}$$



$$Y = \overline{A \cdot B}$$



$$Y = A \oplus B$$



$$Y = \overline{A \oplus B}$$

Question: How many basic one and two input gates exist and how many of these are useful?

# The basic logic gates

The set of NOR gates is complete

Any combinational logic function can be realized with only multiple-input NOR gates

The set of NAND gates is complete

Any combinational logic function can be realized with only multiple-input NAND gates

Performance of the BASIC gates is critical!

# The basic logic gates

A gate logic family can be formed based upon a specific design style for implementing logic functions

Many different gate logic family types exist

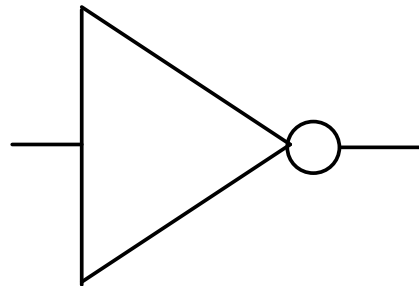
NMOS, PMOS, CMOS, TTL, ECL, RTL, DCTL,...

Substantial differences in performance from one family type to another

Power, Area, Noise Margins, ....

# The basic logic gates

It suffices to characterize the inverter of a logic family and then express the performance of other gates in that family in terms of the performance of the inverter.



What characteristics are required and desirable for an inverter to form the basis for a useful logic family?

# What restrictions are there on the designer for building Boolean circuits?

- None !!!!
- It must “work” as expected
- Designer is Master of the silicon !

# Desirable and/or Required Logic Family Characteristics

**What are the desired characteristics of a logic family?**

# Desirable and/or Required Logic Family Characteristics

1. High and low logic levels must be uniquely distinguishable (even in a long cascade)
2. Capable of driving many loads (good fanout)
3. Fast transition times (but in some cases, not too fast)
4. Good noise margins (low error probabilities)
5. Small die area
6. Low power consumption
7. Economical process requirements



# Desirable and/or Required Logic Family Characteristics

8. Minimal noise injection to substrate
9. Low leakage currents
10. No oscillations during transitions
11. Compatible with synthesis tools
12. Characteristics do not degrade too much with temperature
13. Characteristics do not vary too much with process variations

Are some of these more important than others?

Yes ! – must have well-defined logic levels for circuits to even function as logic

# Desirable and/or Required Logic Family Characteristics

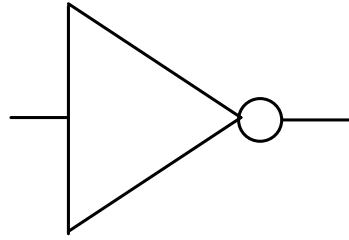
Are some of these more important than others?

Yes ! – must have well-defined logic levels for circuits to even function as logic

What properties of an inverter are necessary for it to be useful for building a logic family

What are the logic levels for a given inverter of a given logic family?

What are the logic levels for a given inverter of for a given logic family?



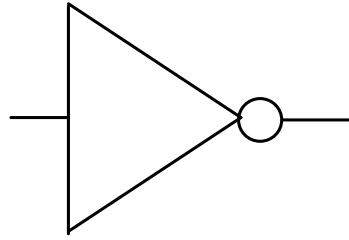
$V_H=?$

$V_L=?$

### Can we legislate them ?

- Some authors choose to simply define a value for them
- Simple and straightforward approach
- **But what if the circuit does not interpret them the same way they are defined !!**

What are the logic levels for a given inverter of for a given logic family?



$V_H=?$

$V_L=?$

**Can we legislate them ?**

In 1897 the Indiana House of Representatives unanimously passed a measure redefining the area of a circle and the value of pi. (House Bill no. 246, introduced by Rep. Taylor I. Record.) The bill died in the state Senate.



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[en.wikipedia.org/wiki/Indiana\\_General\\_Assembly](https://en.wikipedia.org/wiki/Indiana_General_Assembly)

# Indiana Pi Bill

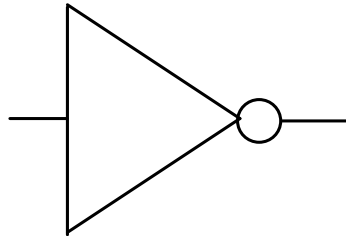
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From Wikipedia, the free encyclopedia

The **Indiana Pi Bill** is the popular name for bill #246 of the 1897 sitting of the [Indiana General Assembly](#), one of the most notorious attempts to establish [mathematical truth](#) by [legislative fiat](#). Despite its name, the main result claimed by the bill is a method to [square the circle](#), although it does imply various incorrect values of the [mathematical constant](#)  $\pi$ , the ratio of the [circumference](#) of a circle to its [diameter](#).<sup>[1]</sup>

The bill, written by a physician who was an amateur mathematician, never became law due to the

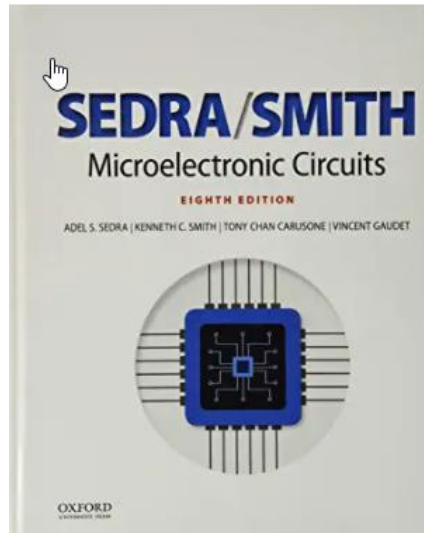
# What are the logic levels for a given inverter of for a given logic family?



$$V_H = ?$$

$$V_L = ?$$

Can we legislate them ?



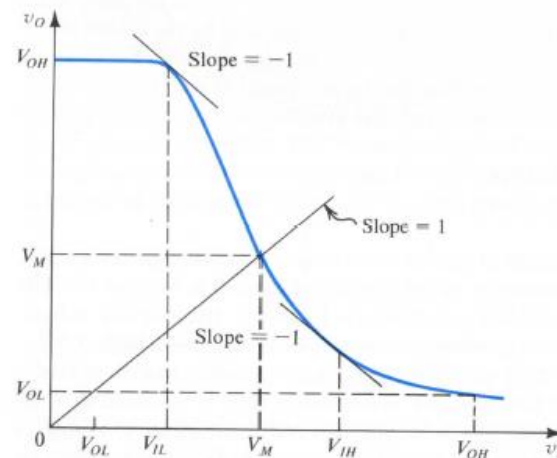
World's most widely used electronics text

**Noise Margins** The static operation of a logic-circuit family is characterized by the voltage transfer characteristic (VTC) of its basic inverter. Figure 10.2 shows such a VTC and defines its four parameters;  $V_{OH}$ ,  $V_{OL}$ ,  $V_{IH}$ , and  $V_{IL}$ . Note that  $V_M$  and  $V_M$  are defined as the points at which the slope of the VTC is  $-1$ . Also indicated is the definition of the threshold voltage  $V_M$ , or  $V_{th}$  as we shall frequently call it, as the point at which  $v_O = v_I$ . Recall that we discussed the VTC in its generic form in Section 1.7, and have also seen actual VTCs: in Section 4.10 for the CMOS inverter, and in Section 5.10 for the BJT inverter.

The **robustness** of a logic-circuit family is determined by its ability to reject noise, and thus by the noise margins  $NH_H$  and  $NM_L$ ,

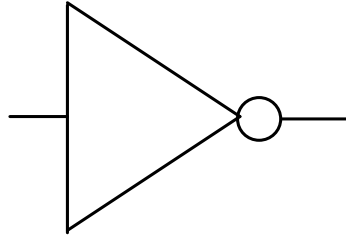
$$NM_H \equiv V_{OH} - V_{IH} \quad (10.1)$$

$$NM_L \equiv V_{IL} - V_{OL} \quad (10.2)$$



**FIGURE 10.2** Typical voltage transfer characteristic (VTC) of a logic inverter, illustrating the definition of the critical points.

# What are the logic levels for a given inverter of for a given logic family?



$$V_H = ?$$

$$V_L = ?$$

Can we legislate them ?

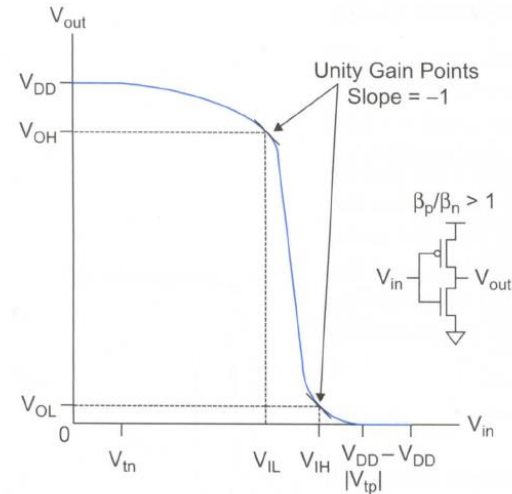
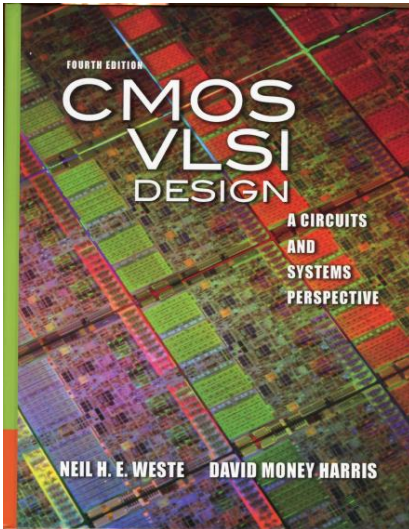
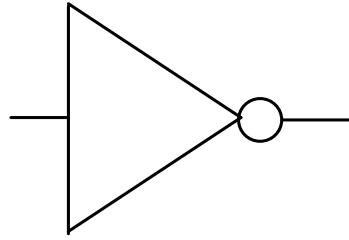


FIGURE 2.30 CMOS inverter noise margins

What are the logic levels for a given inverter of for a given logic family?

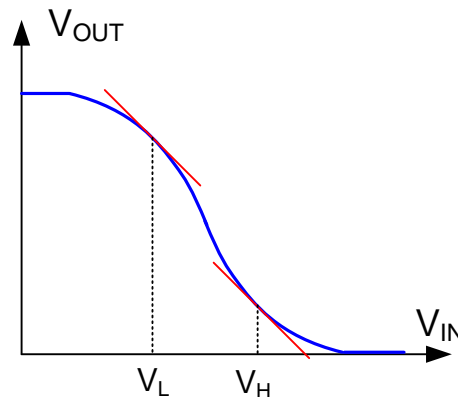


$V_H=?$

$V_L=?$

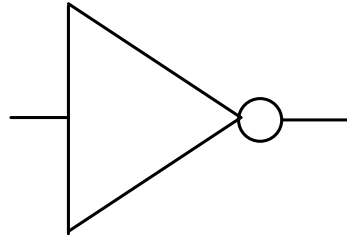
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- Analytical expressions may be complicated
- But what if the circuit does not interpret them the same way they are defined !!





What are the logic levels for a given inverter of for a given logic family?

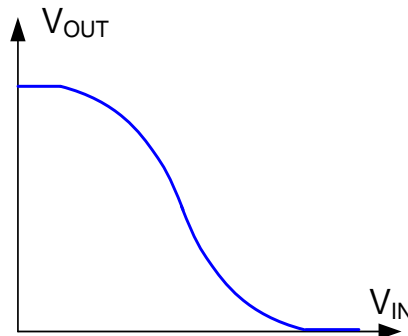


$V_H=?$

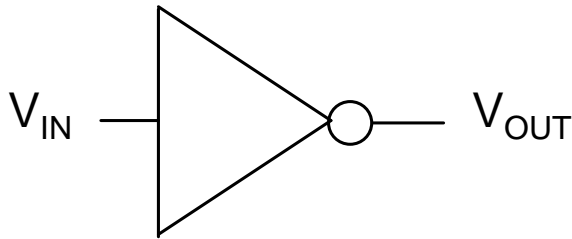
$V_L=?$

Ask the inverter how it will interpret logic levels

- The inverter will interpret them the way the circuit really operate as a Boolean system !!
- Analytical expressions may be complicated
- How is this determination made?

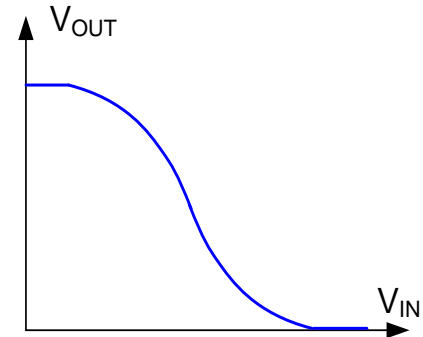


# Ask the inverter how it will interpret logic levels



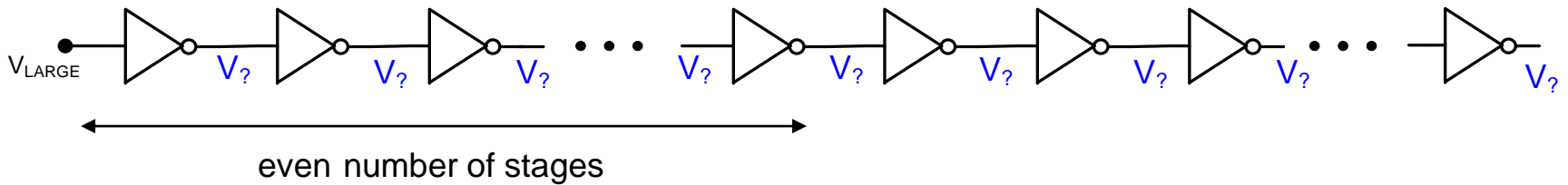
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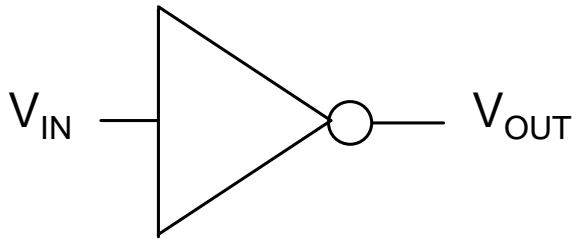


Consider a very long cascade of inverters

Apply a large voltage at the input (alternatively a small input could be used)  
w.l.o.g. assume an even number of inverters in chain indicated

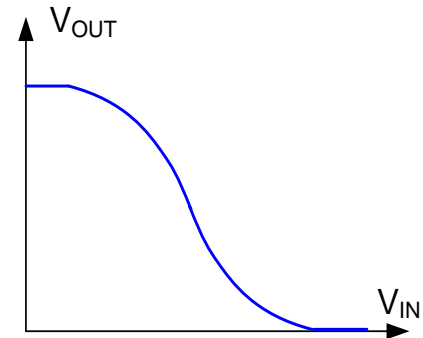


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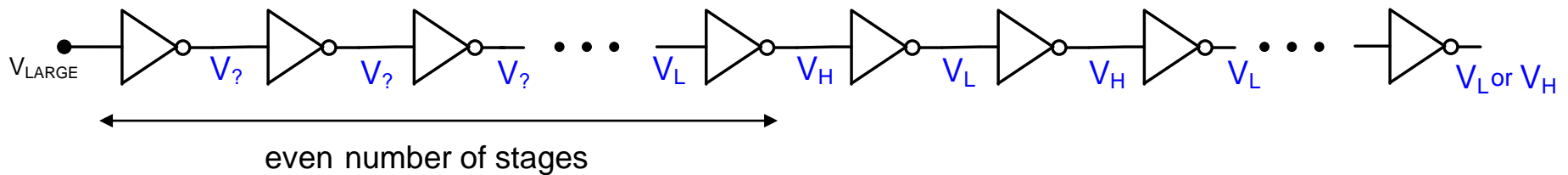
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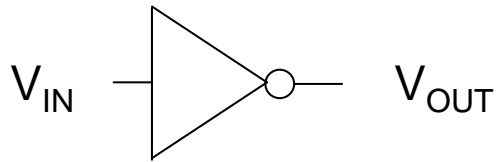
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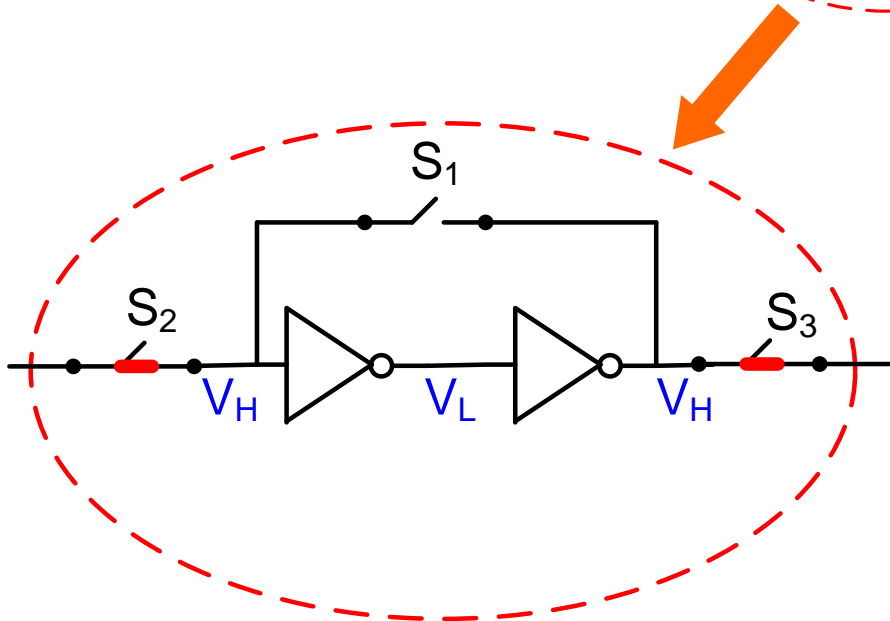
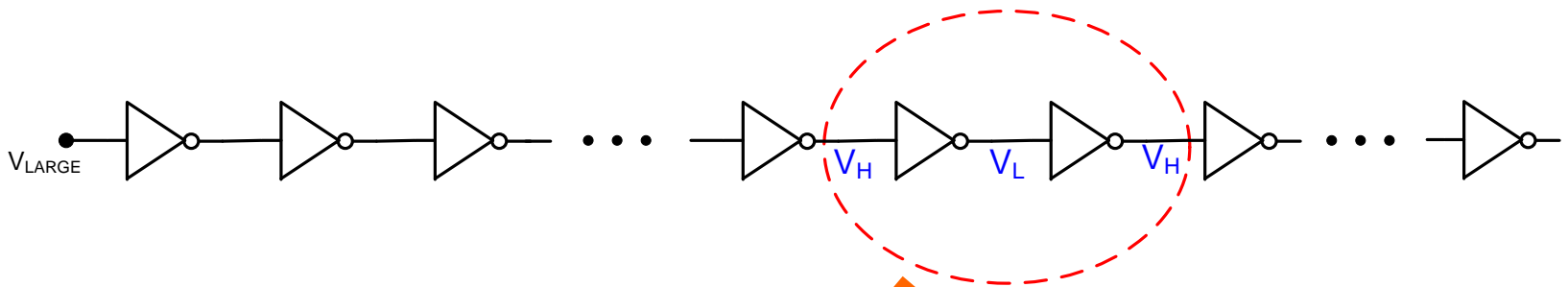
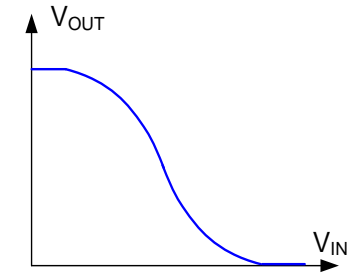
If logic levels are to be maintained, the voltage at the end of this even number of stages must be  $V_H$ , that of the next must be  $V_L$ , the next  $V_H$ , etc. until the start of the cascade is approached

# Ask the inverter how it will interpret logic levels

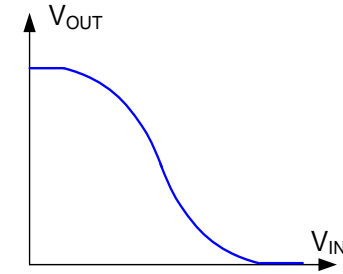
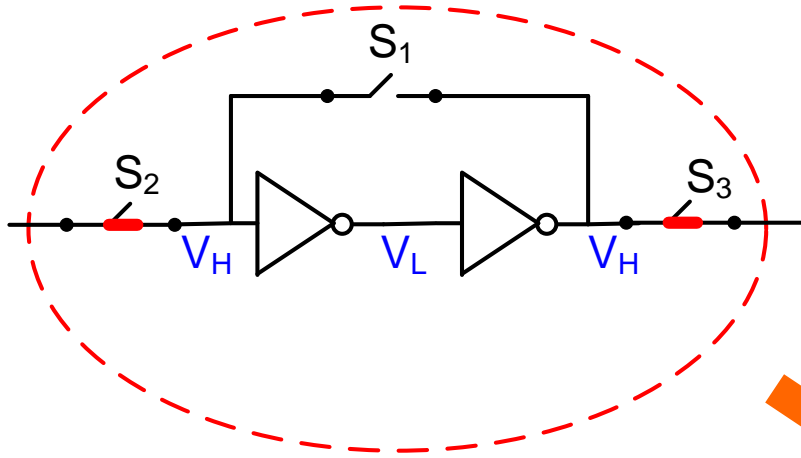


$V_H=?$

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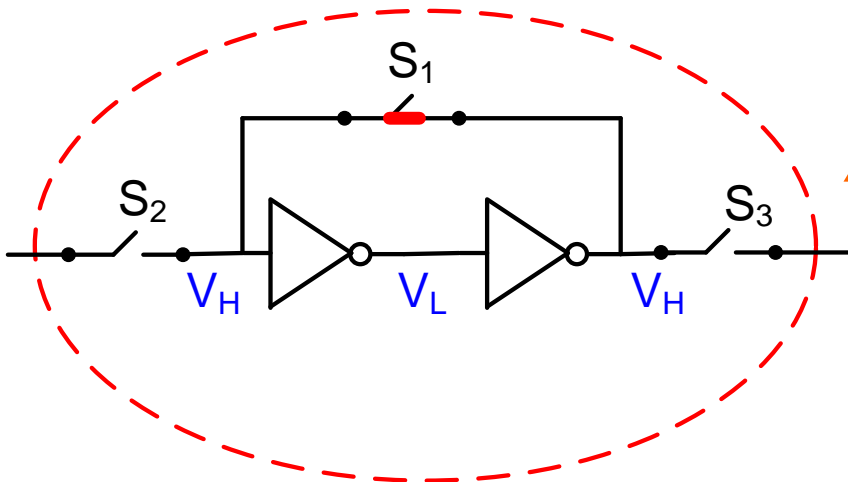
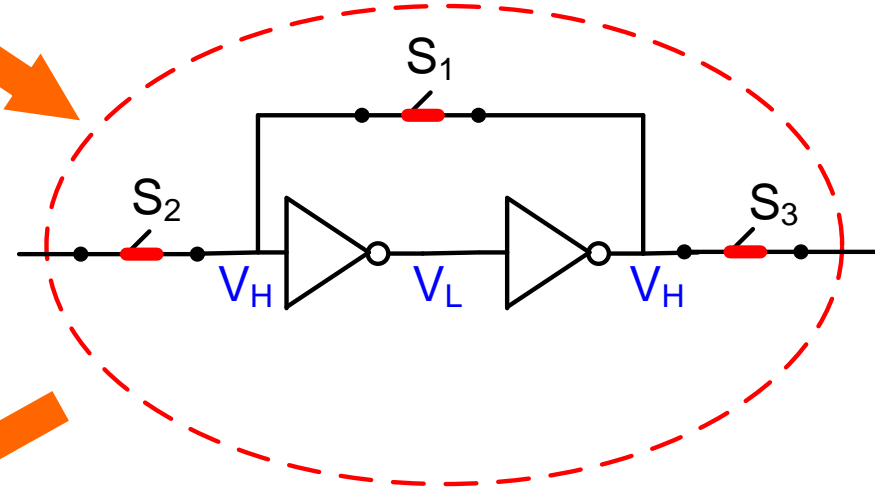


# Ask the inverter how it will interpret logic levels

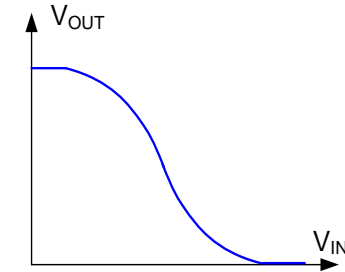
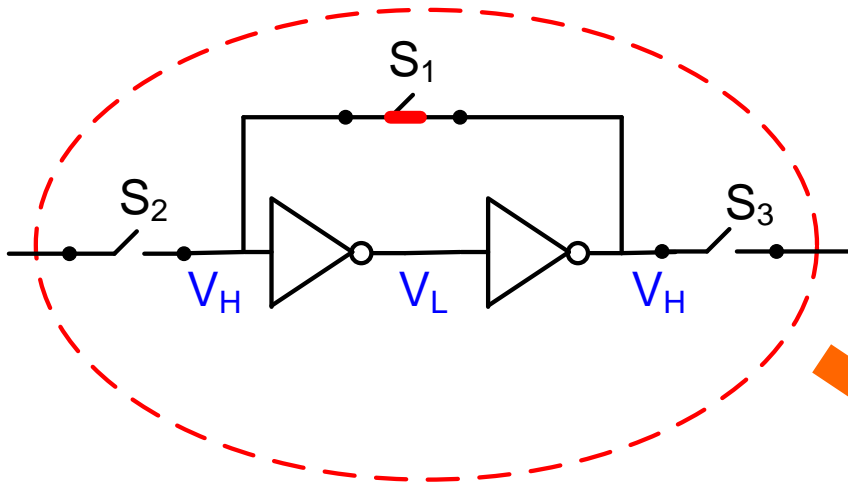


$V_H=?$

$V_L=?$

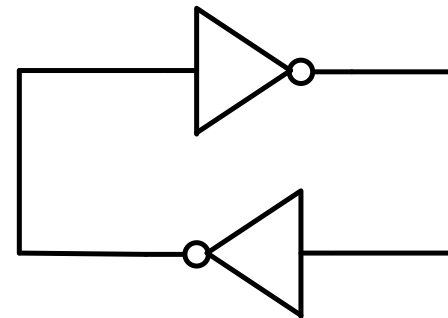


# Ask the inverter how it will interpret logic levels



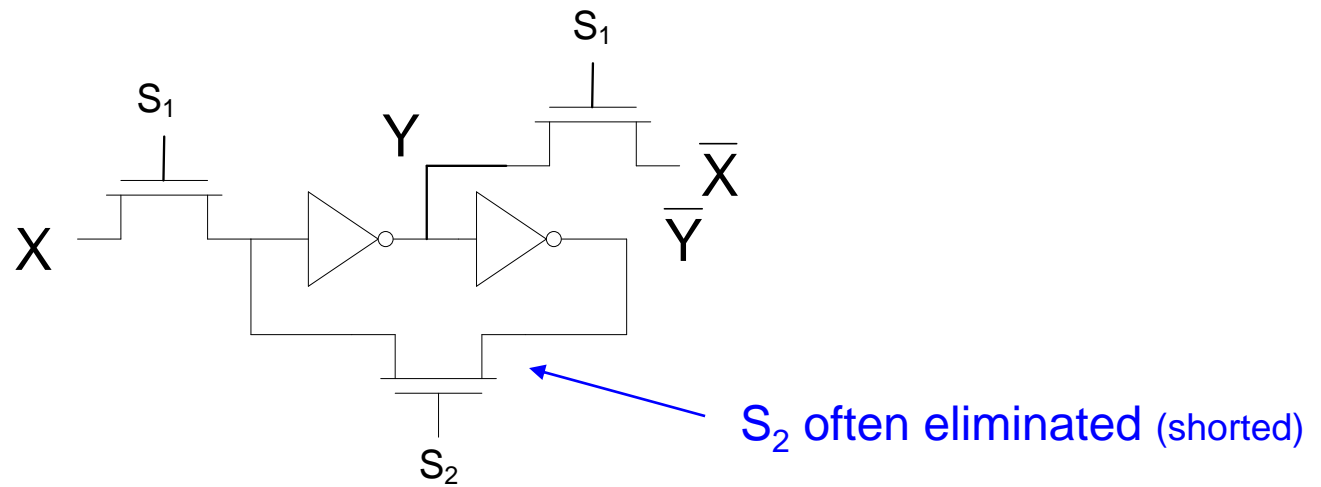
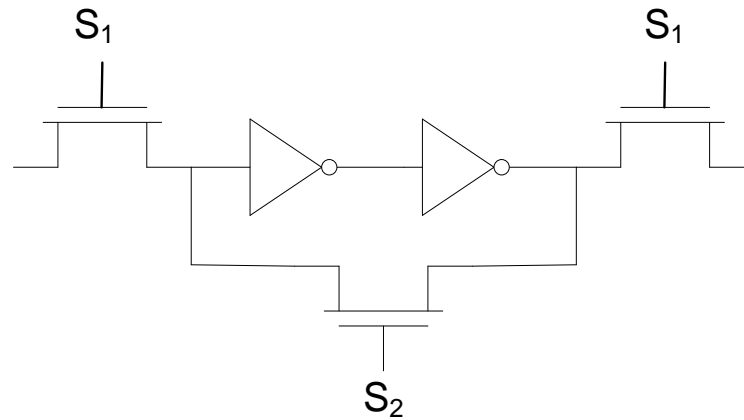
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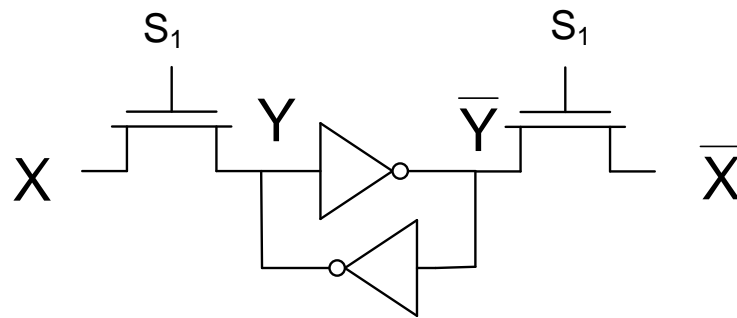
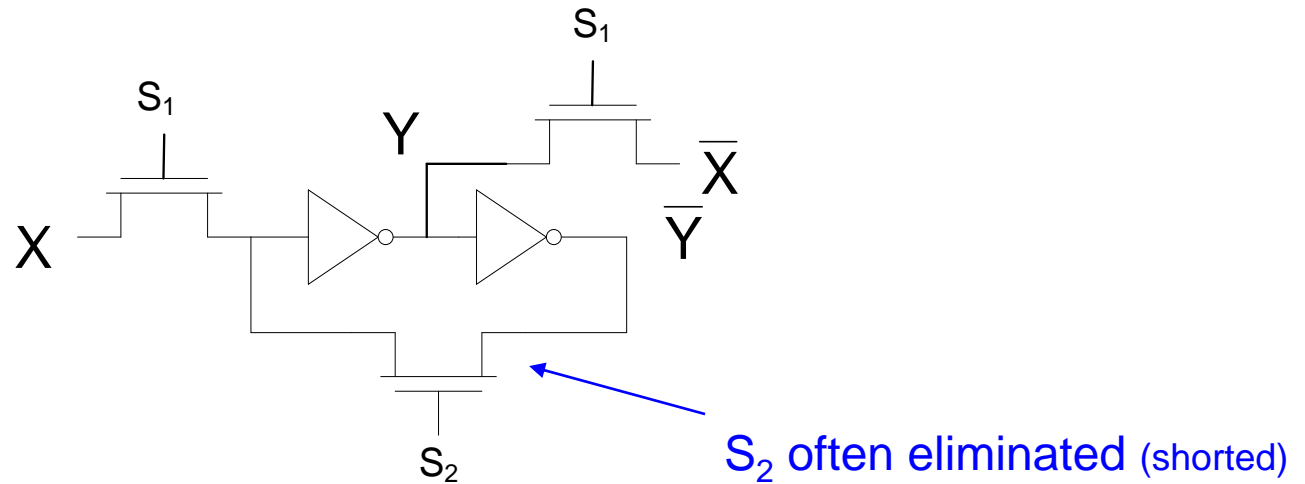
- Two inverter loop
- Very useful circuit !

# The two-inverter loop



SRAM Cell

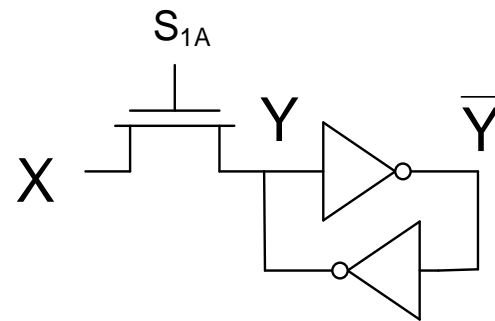
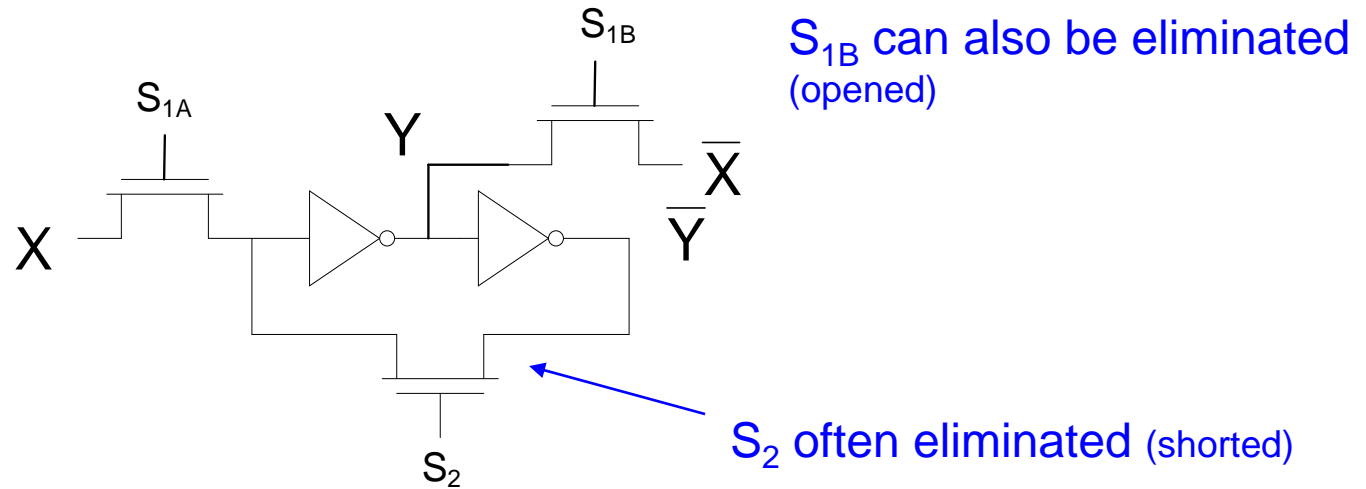
# The two-inverter loop



Standard 6-transistor SRAM Cell



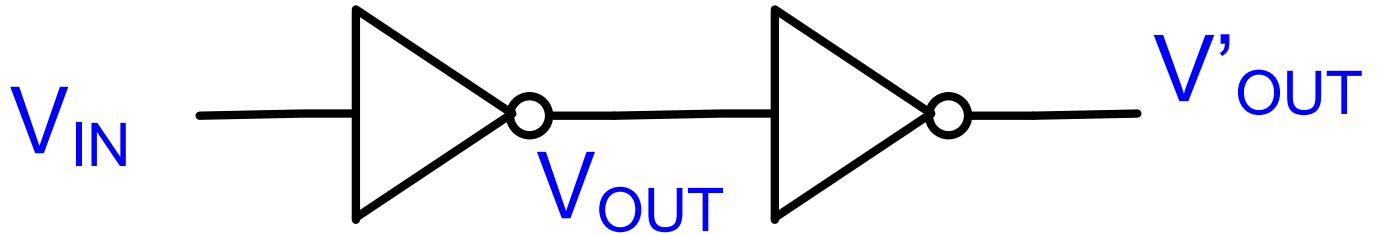
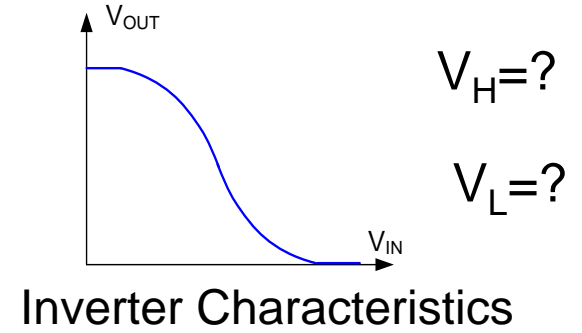
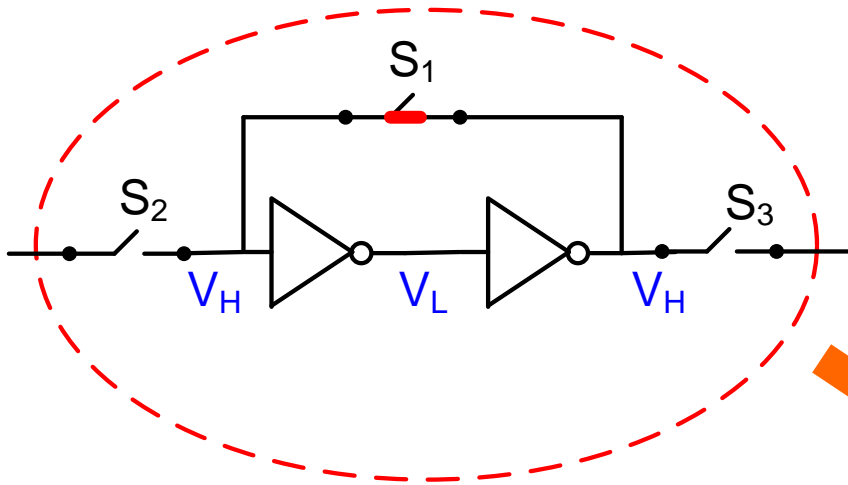
# The two-inverter loop



5-transistor SRAM Cell

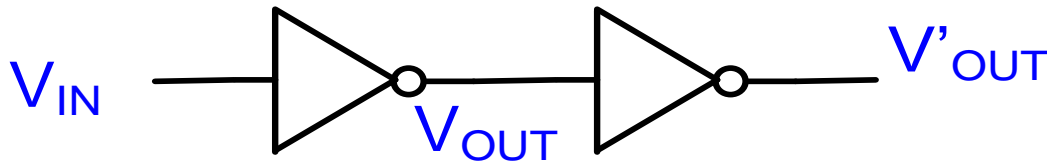
Will also work but less common (less area but degraded performance)

Ask the inverter how it will interpret logic levels

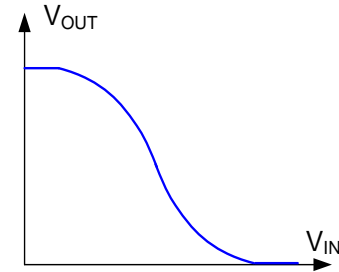


Thus, consider the inverter pair

# Ask the inverter how it will interpret logic levels



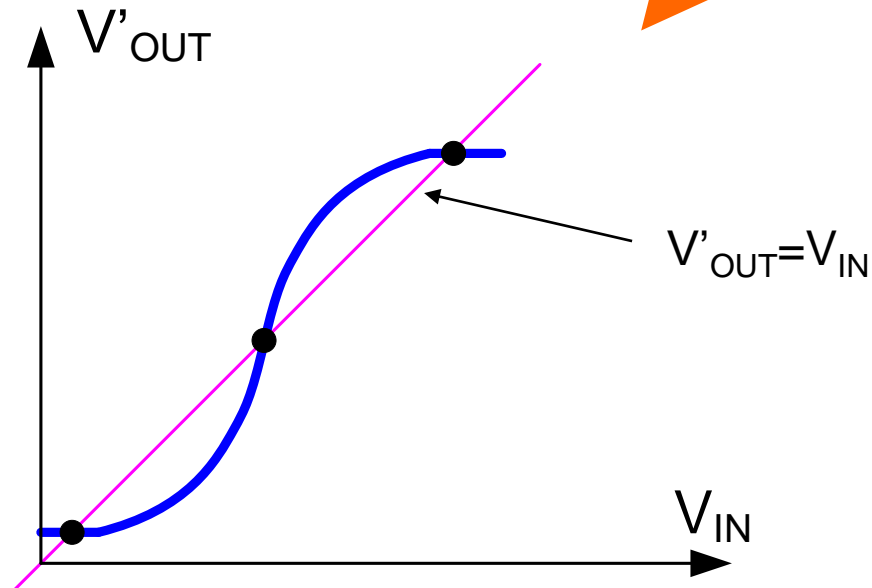
Inverter pair



$V_H=?$

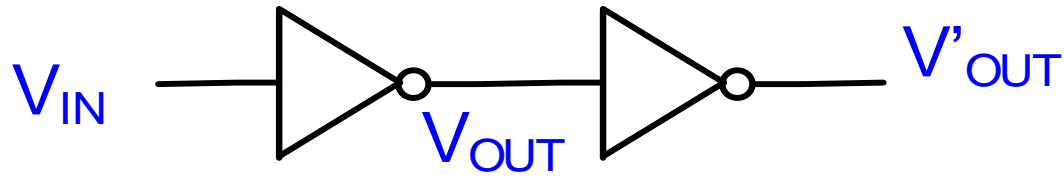
$V_L=?$

$V_H$  and  $V_L$  will be on the intersection of the transfer characteristics of the inverter pair (IPTC) and the  $V'_{OUT}=V_{IN}$  line

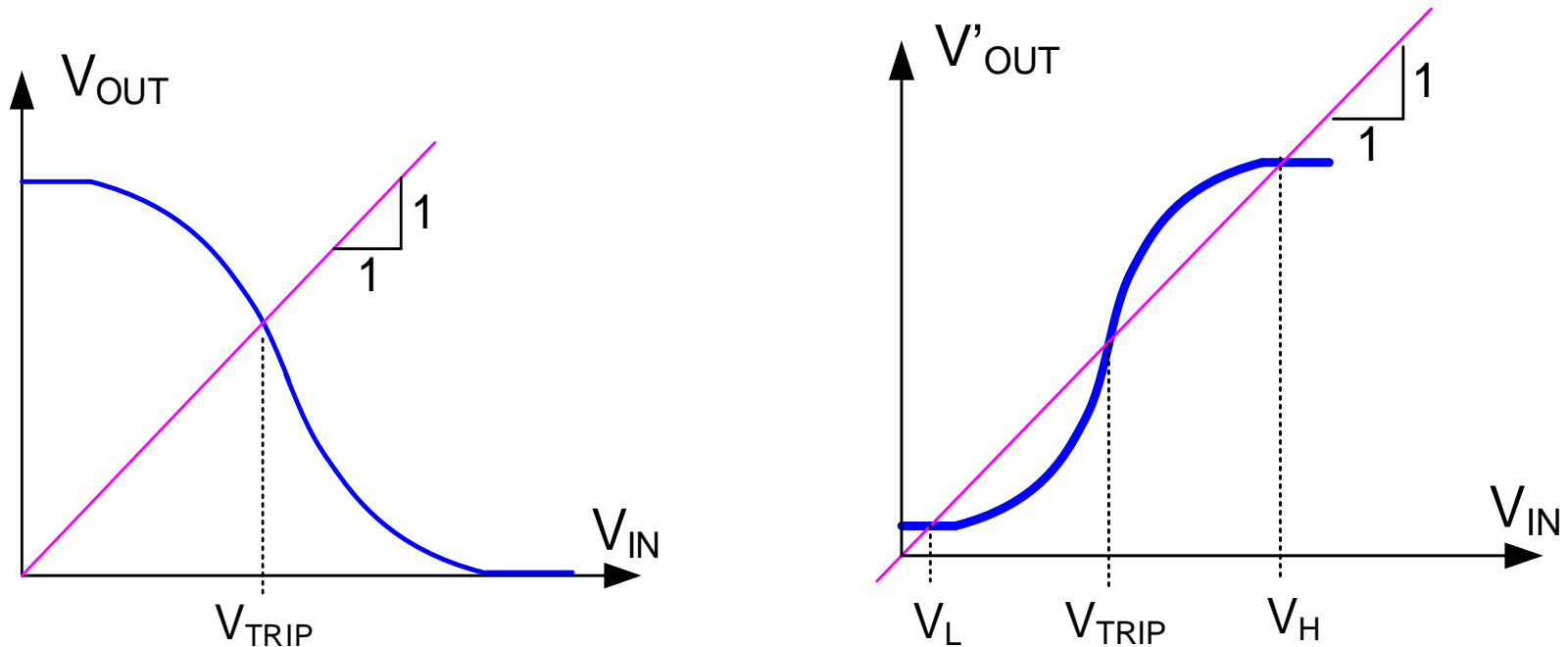


$V_H$  and  $V_L$  often termed the "1" and "0" states

## Observation



When  $V_{OUT}=V_{IN}$  for the inverter,  $V'_{OUT}$  is also equal to  $V_{IN}$ . Thus the intersection point for  $V_{OUT}=V_{IN}$  in the inverter transfer characteristics (ITC) is also an intersection point for  $V'_{OUT}=V_{IN}$  in the inverter-pair transfer characteristics (IPTC)



**Implication:** Inverter characteristics can be used directly to obtain  $V_{TRIP}$

# Logic Family Characteristics

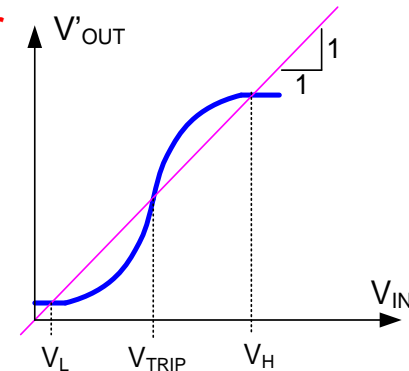
What properties of an inverter are necessary for it to be useful for building a two-level logic family?

The inverter-pair transfer characteristics must have three unique intersection points with the  $V'_{OUT} = V_{IN}$  line

What are the logic levels for a given inverter or for a given logic family?

The two extreme intersection points of the inverter-pair transfer characteristics with the  $V'_{OUT} = V_{IN}$  line

Can we legislate  $V_H$  and  $V_L$  for a logic family? **No!**

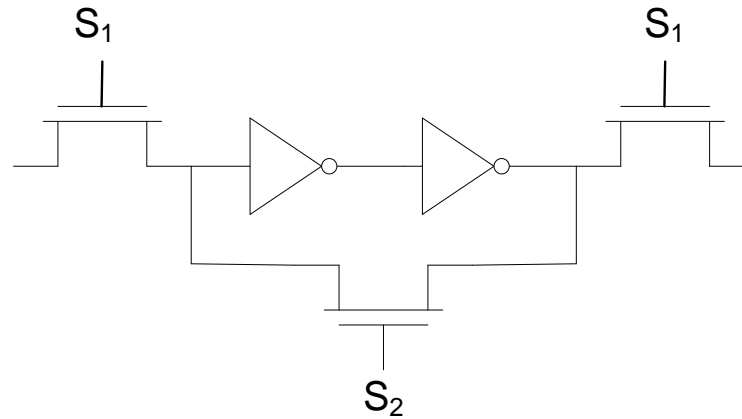


What other properties of the inverter are desirable?

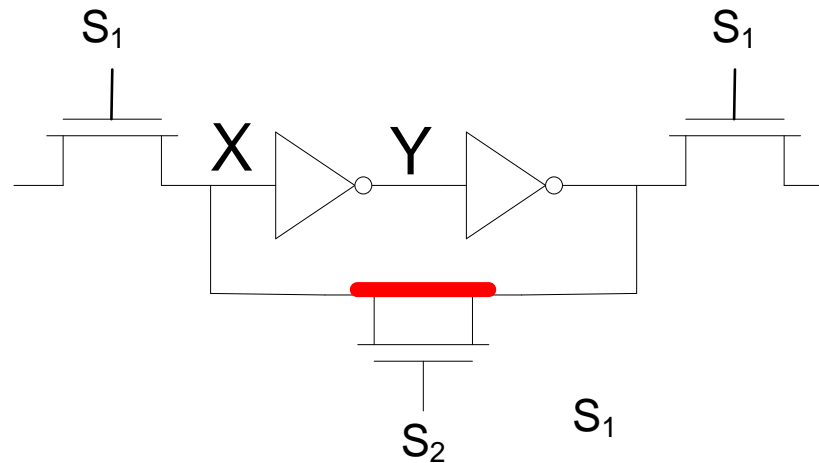
Reasonable separation between  $V_H$  and  $V_L$  (enough separation so that noise does not cause circuit to interpret level incorrectly)

$$V_{TRIP} \approx \frac{V_H + V_L}{2} \quad (\text{to provide adequate noise immunity and process insensitivity})$$

# What happens near the quasi-stable operating point?

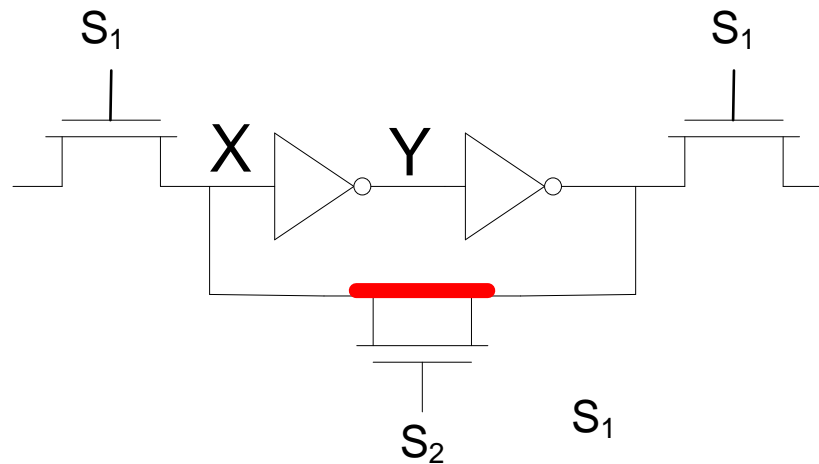


$S_2$  closed and  $X=Y=V_{TRIP}$



# What happens near the quasi-stable operating point?

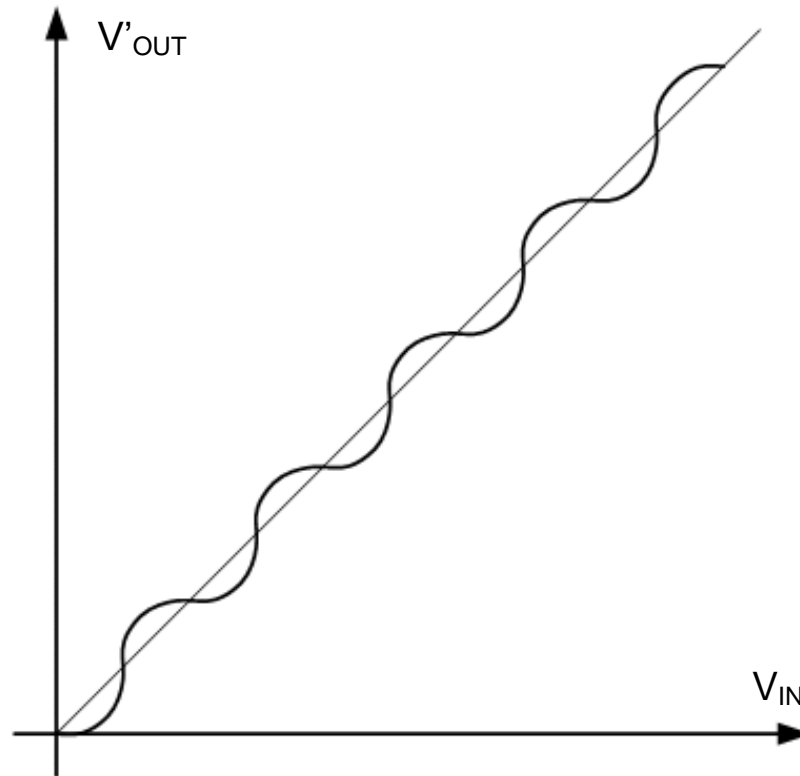
$S_2$  closed and  $X=Y=V_{TRIP}$



If  $X$  decreases even very slightly, will move to the  $X=0, Y=1$  state (very fast)

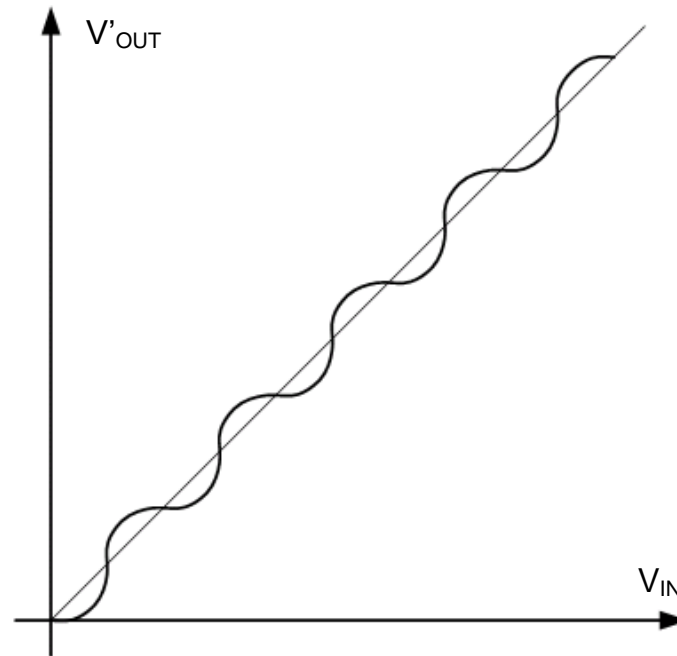
If  $X$  increases even very slightly, will move to the  $X=1, Y=0$  state (very fast)

What if the inverter pair had the following transfer characteristics?





# What if the inverter pair had the following transfer characteristics?

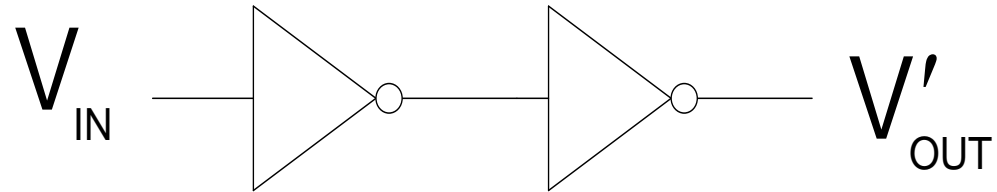


Multiple levels of logic

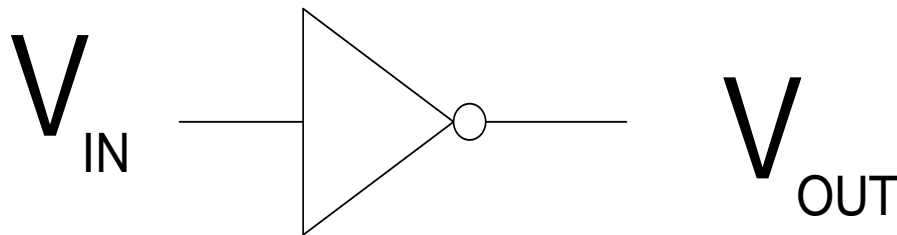
Every intersection point with slope  $< 1$  is a stable point

Every intersection point with slope  $> 1$  is a quasi-stable point

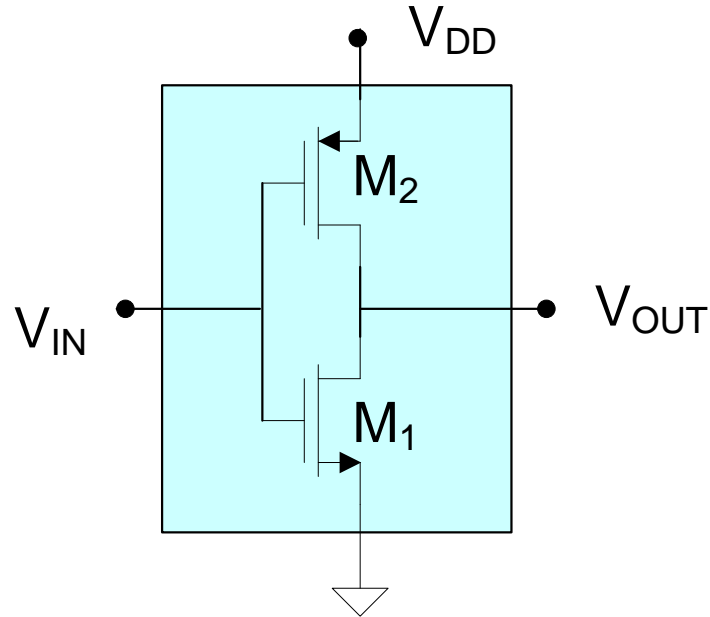
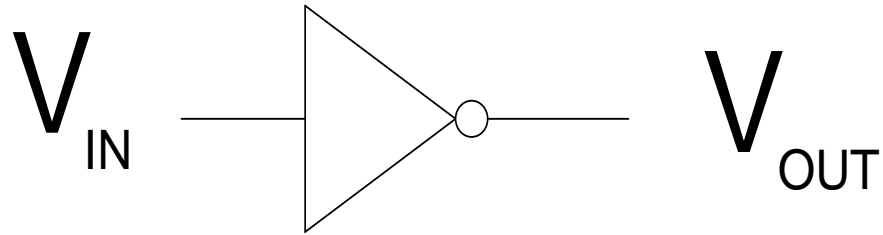
# What are the transfer characteristics of the static CMOS inverter pair?

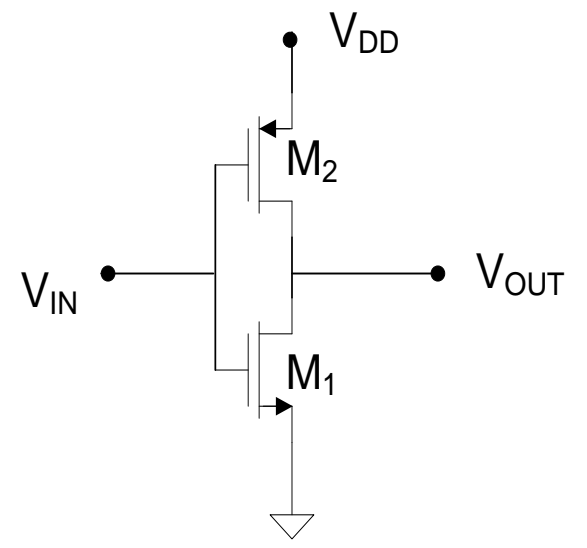


Consider first the inverter



# Transfer characteristics of the static CMOS inverter





# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $V_{IN}$  is so high that  $M_1$  triode,  $M_2$  cutoff

$$I_{D1} = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

$$I_{D2} = 0$$

Equating  $I_{D1}$  and  $-I_{D2}$  we

obtain:

$$0 = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

It can be shown that setting the first product term to 0 will not verify, thus

$$V_{OUT} = 0$$

valid for:

$$V_{GS1} \geq V_{Tn}$$

$$V_{DS1} < V_{GS1} - V_{Tn}$$

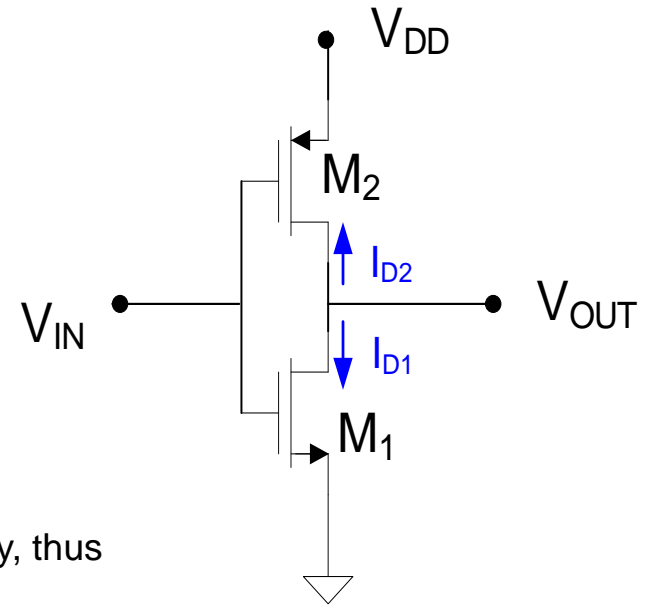
$$V_{GS2} \geq V_{Tp}$$

thus, valid for:

$$V_{IN} \geq V_{Tn}$$

$$V_{OUT} < V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \geq V_{Tp}$$



# Graphical Interpretation of these conditions:

$$V_{IN} \geq V_{Tn}$$

$$V_{OUT} < V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \geq V_{Tp}$$

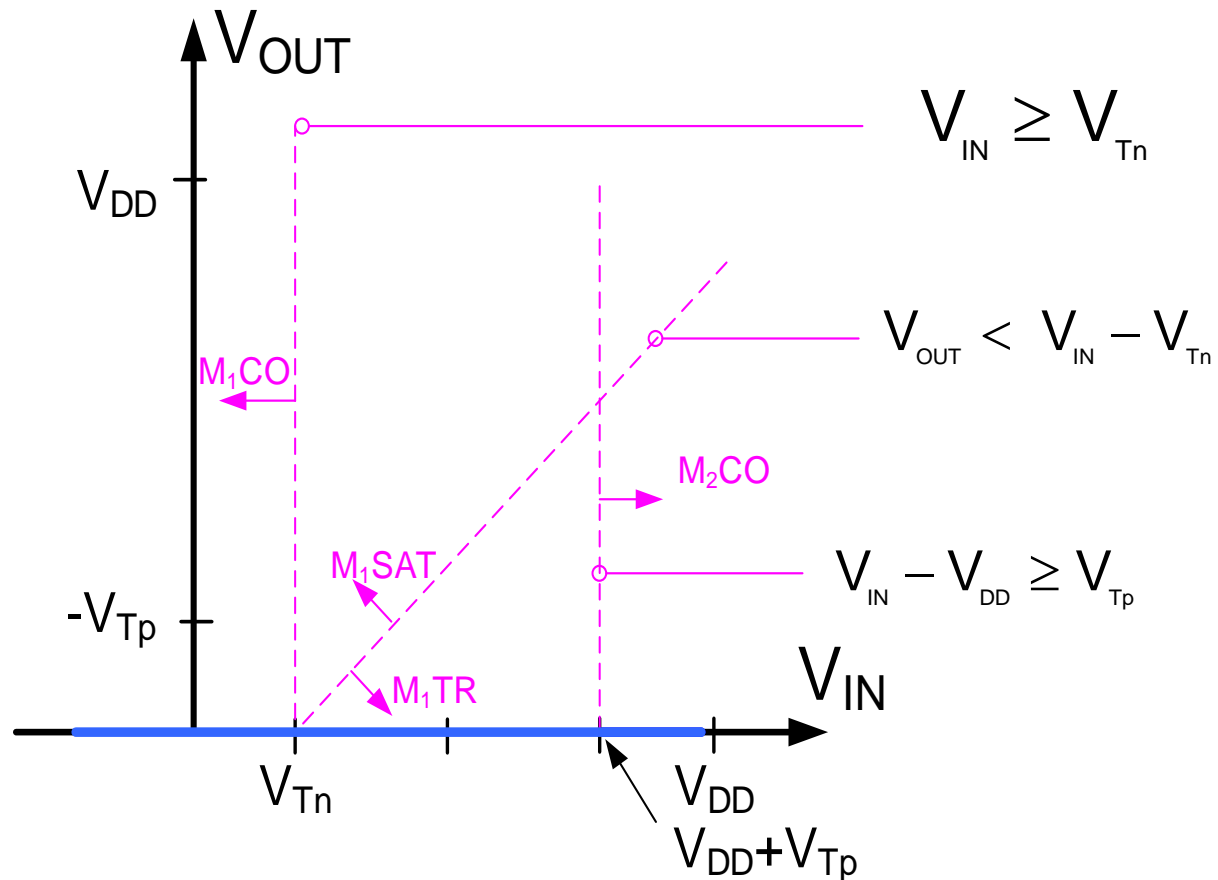


# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $M_1$  triode,  $M_2$  cutoff

$$V_{\text{OUT}} = 0$$

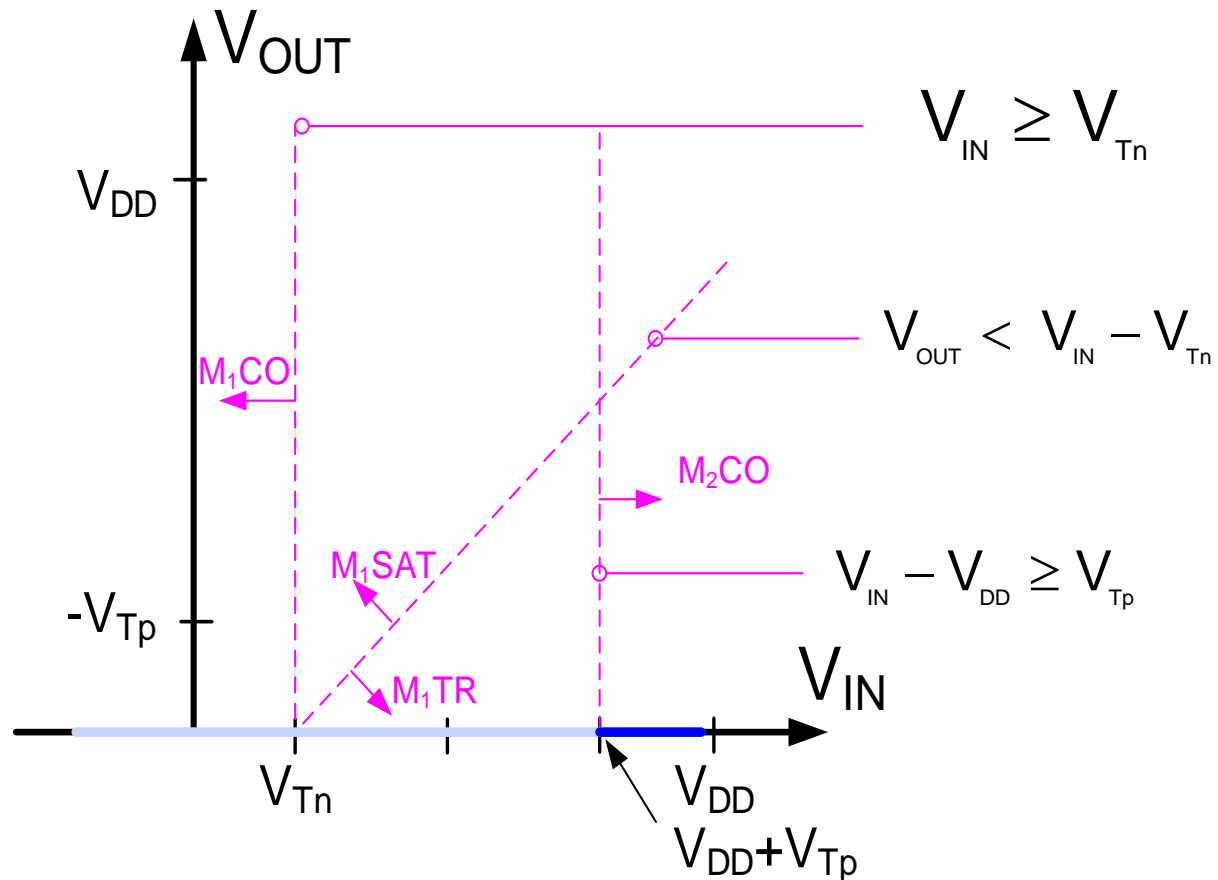


# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $M_1$  triode,  $M_2$  cutoff

$$V_{OUT} = 0$$

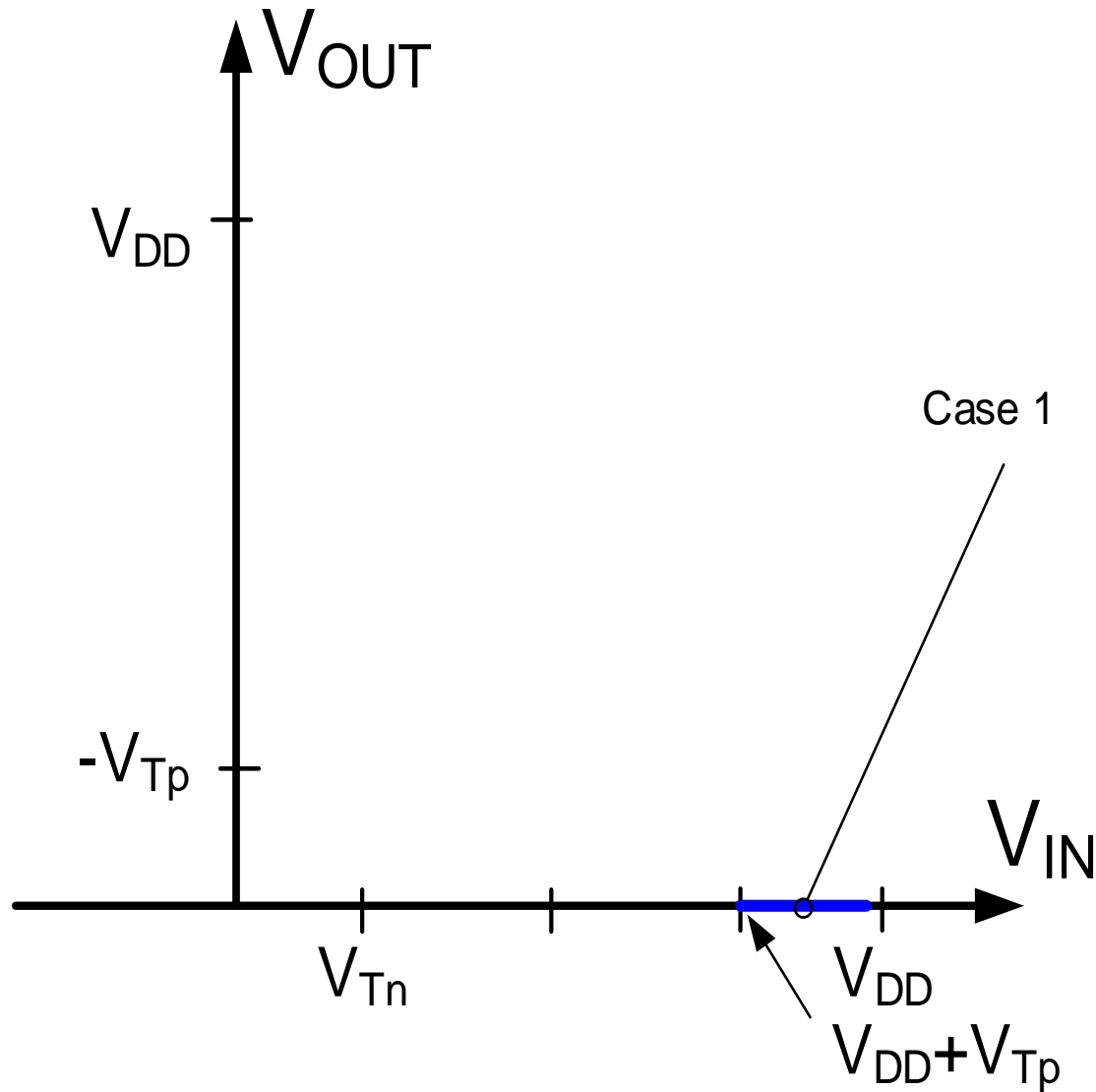




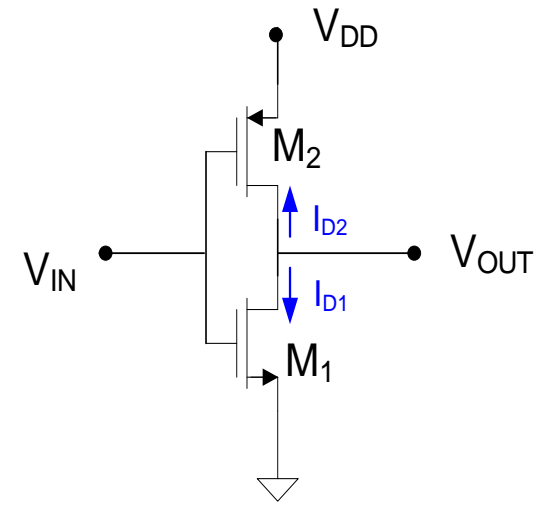
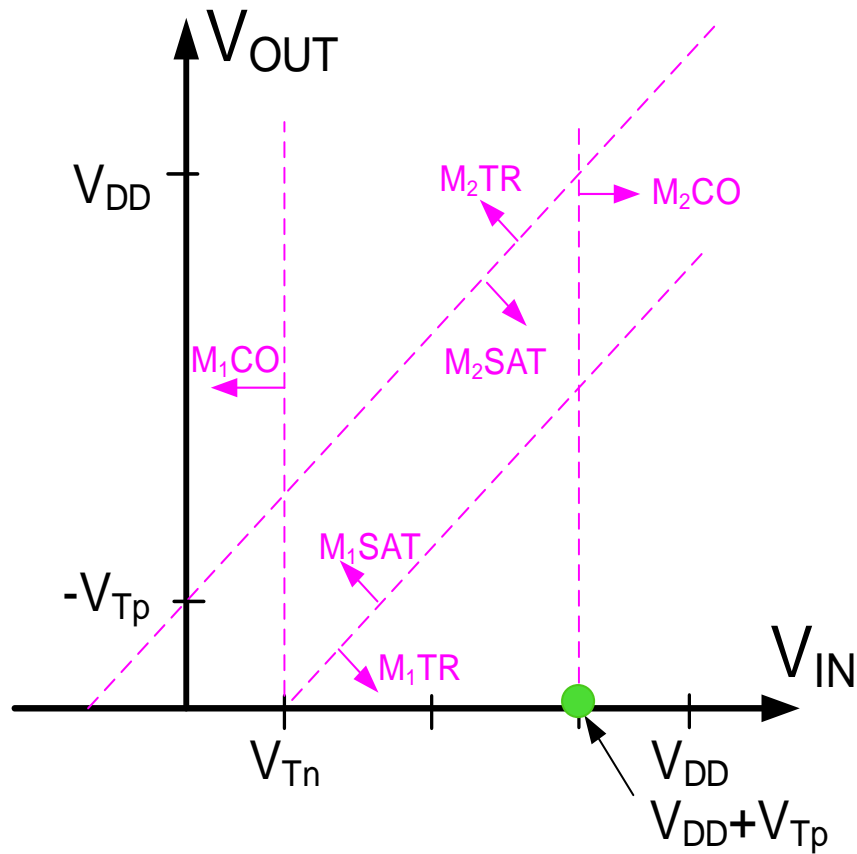
# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Partial solution:



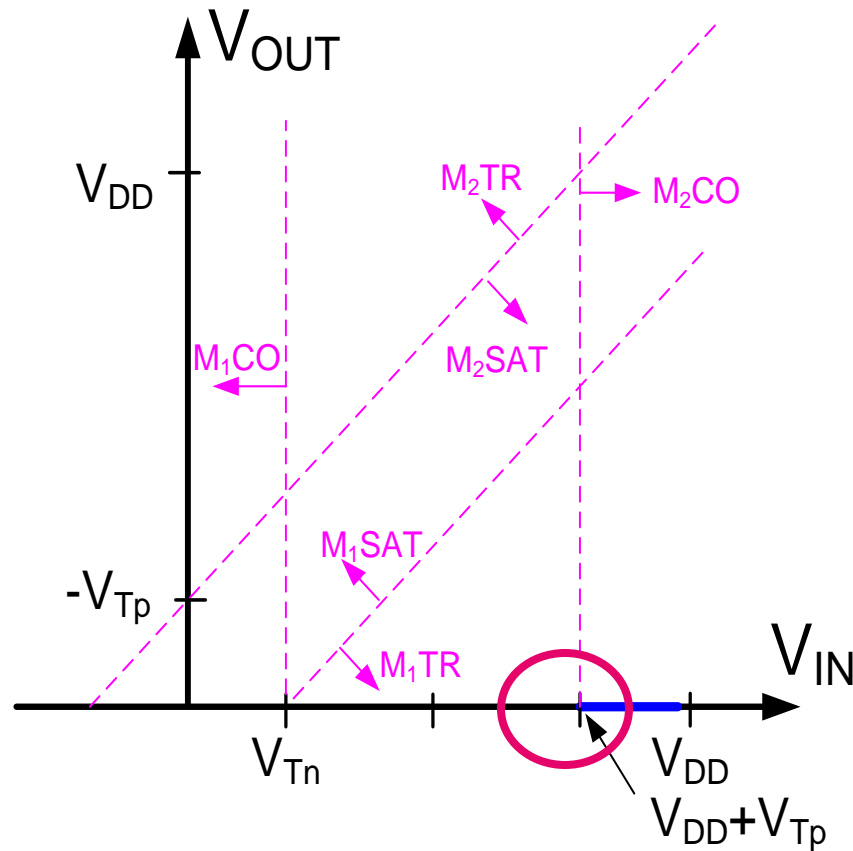
# Regions of Operation for Devices in CMOS inverter



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat



$M_2$ : Square law  $I_D$

$M_1$ : like a resistor

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat

$$I_{D1} = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

$$I_{D2} = -\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

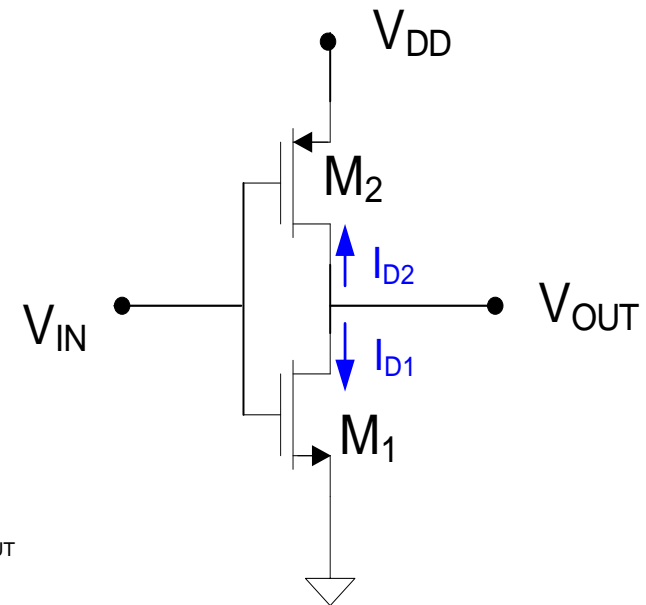
$$\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2 = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} < V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} \leq V_{GS2} - V_{T2}$$

thus, valid for:

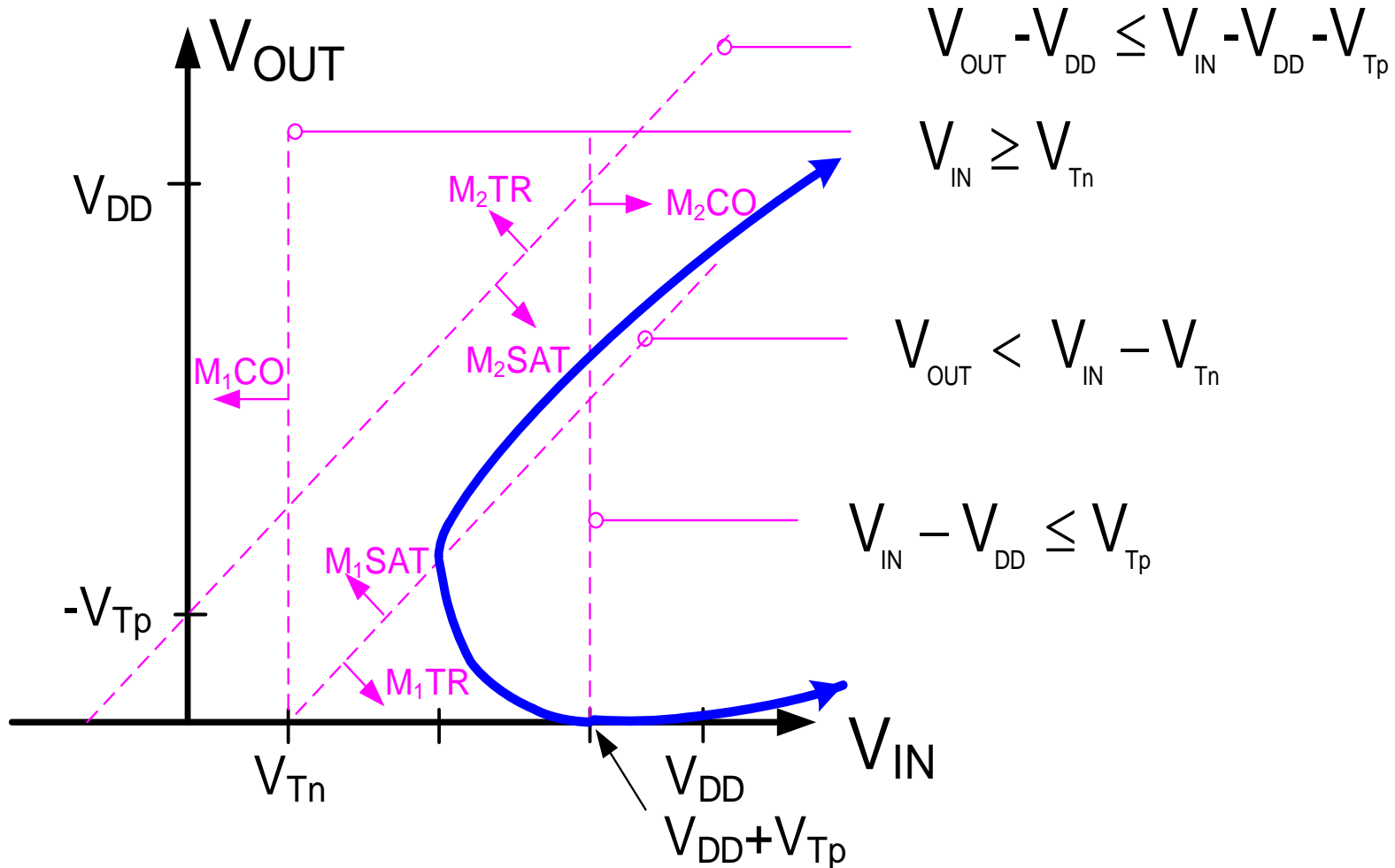
$$V_{IN} \geq V_{Tn} \quad V_{OUT} < V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} \leq V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

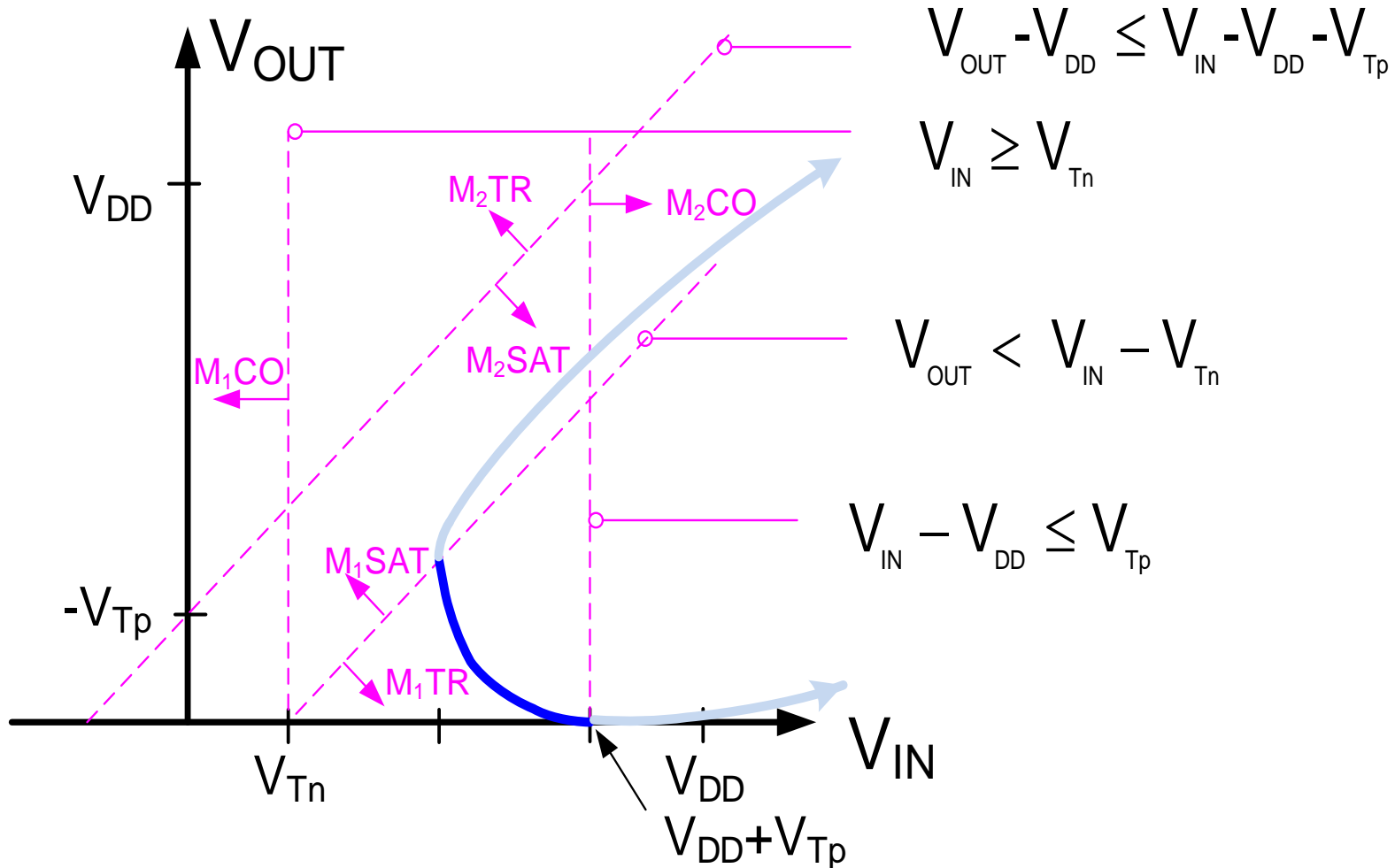
Case 2  $M_1$  triode,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

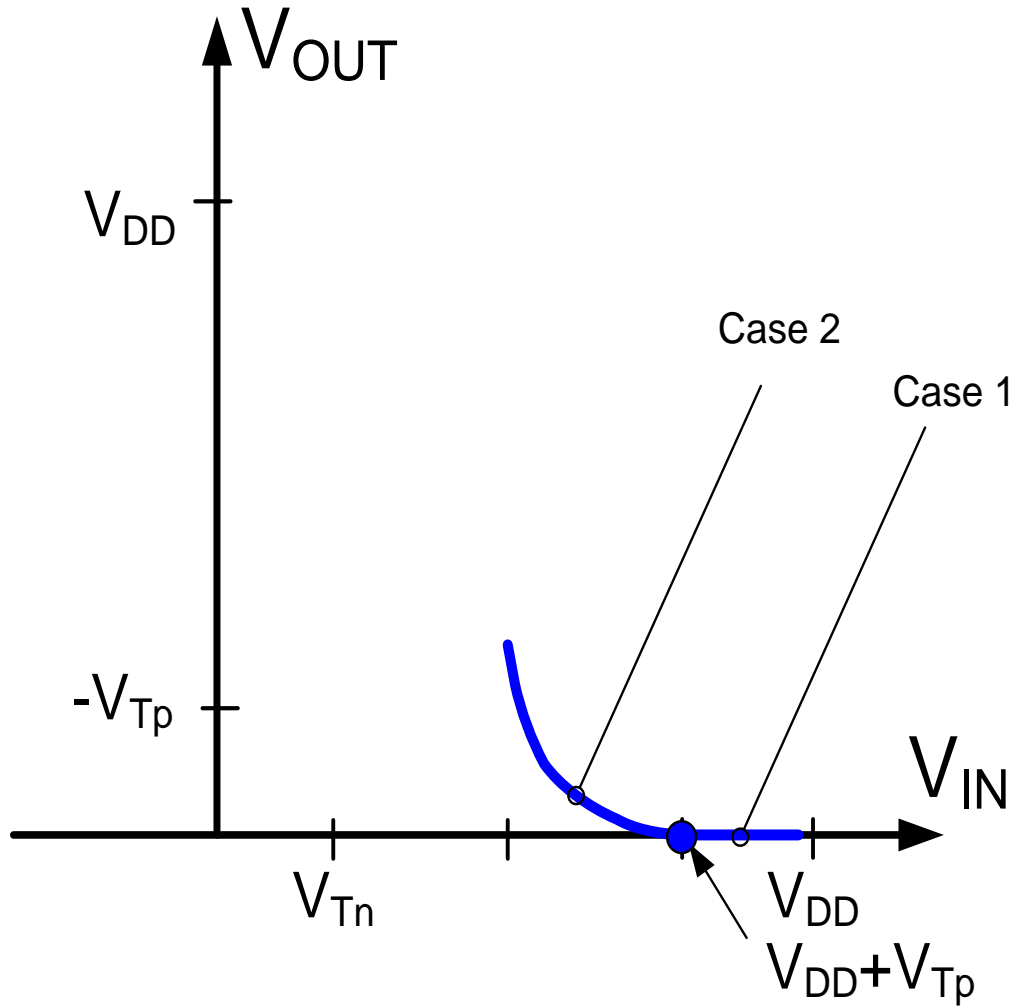
(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

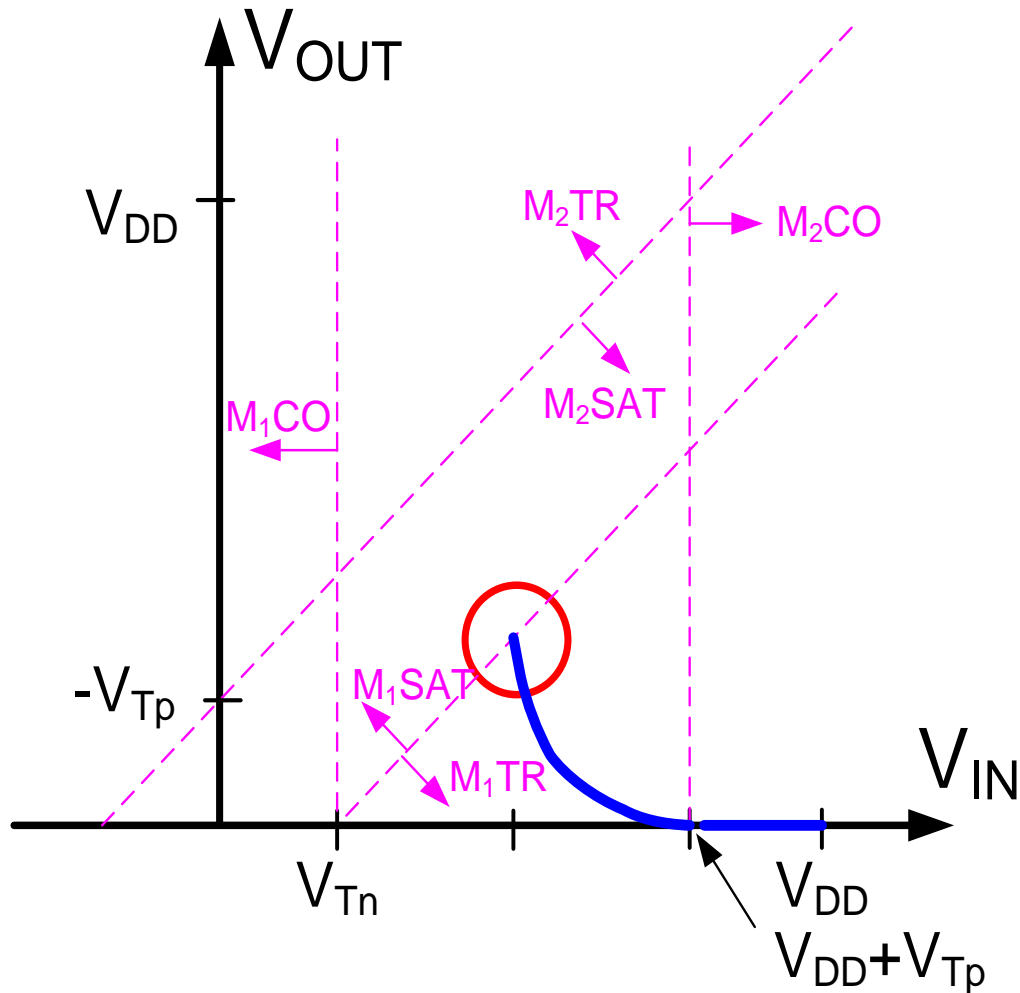
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat





# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat

$$I_{D1} = \frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

$$I_{D2} = \frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

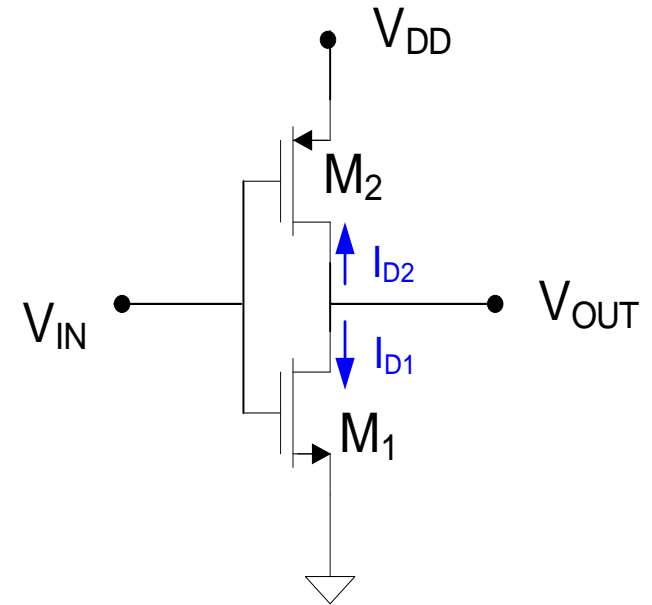
$$\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2 = \frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

Which can be rewritten as:

$$\sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}} (V_{DD} + V_{Tp} - V_{IN}) = \sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} (V_{IN} - V_{Tn})$$

Which can be simplified to:

$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}}}{\sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} + \sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}}}$$



This is a vertical line

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat

$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2}}}{\sqrt{\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1}} + \sqrt{\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2}}}$$

Since  $C_{oxn} \cong C_{oxp} = C_{ox}$  this can be simplified to:

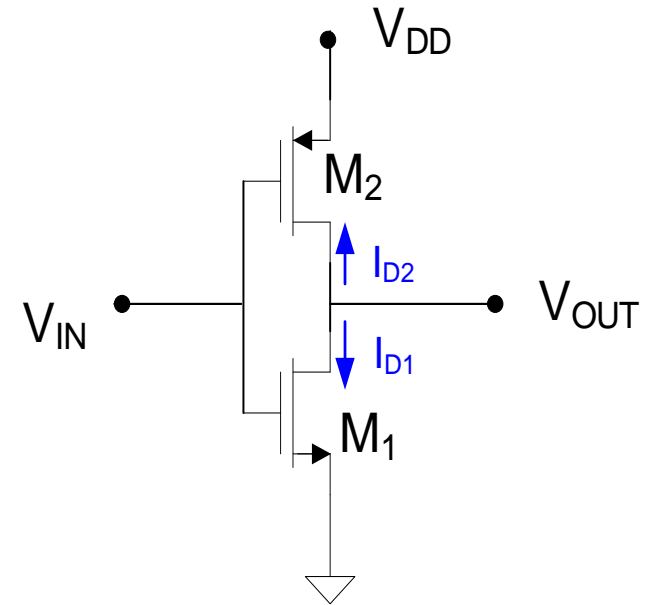
$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{L_2}}}{\sqrt{\frac{W_1}{L_1}} + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{L_2}}}$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} \geq V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} \leq V_{GS2} - V_{T2}$$

thus, valid for:

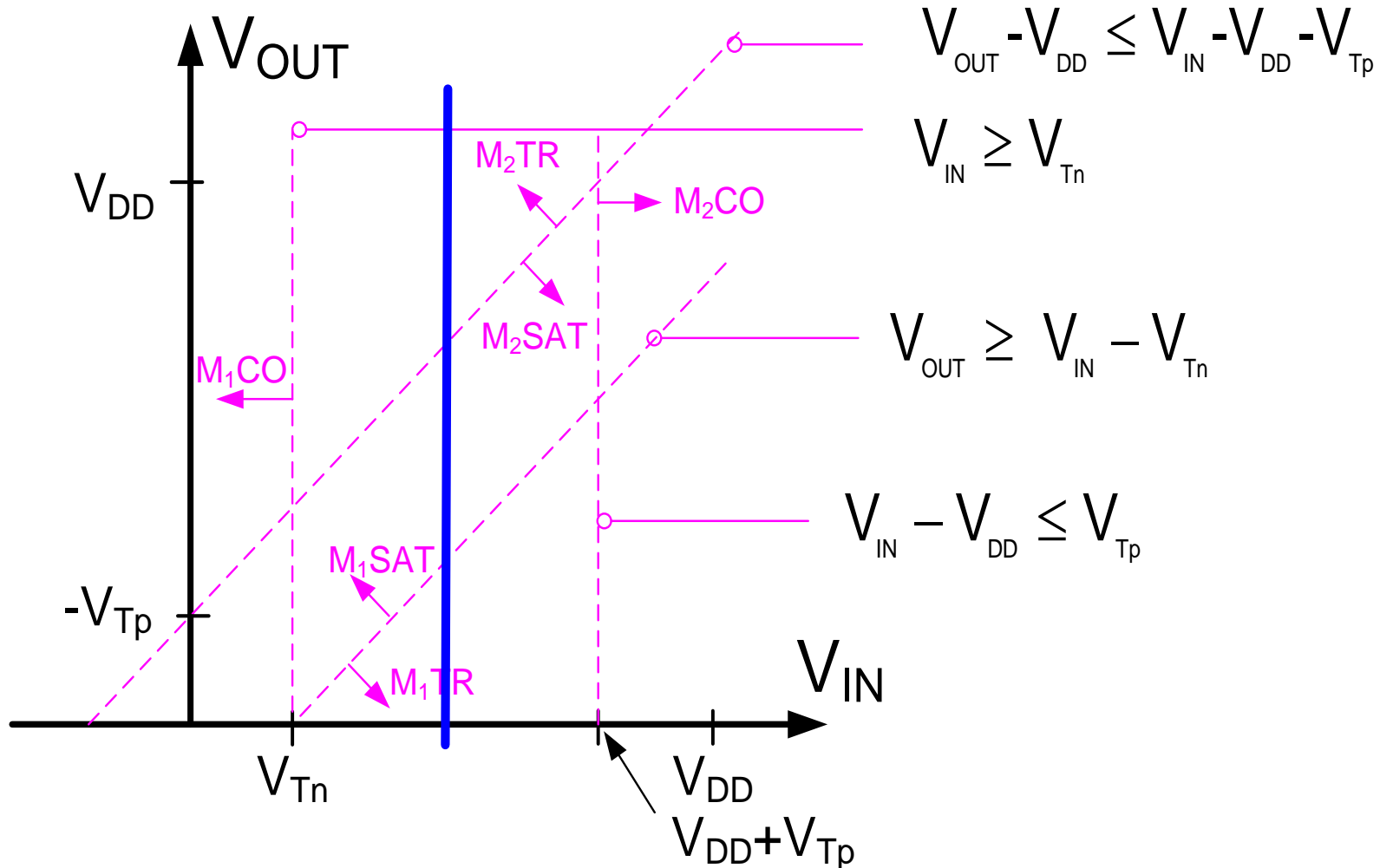
$$V_{IN} \geq V_{Tn} \quad V_{OUT} \geq V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} \leq V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

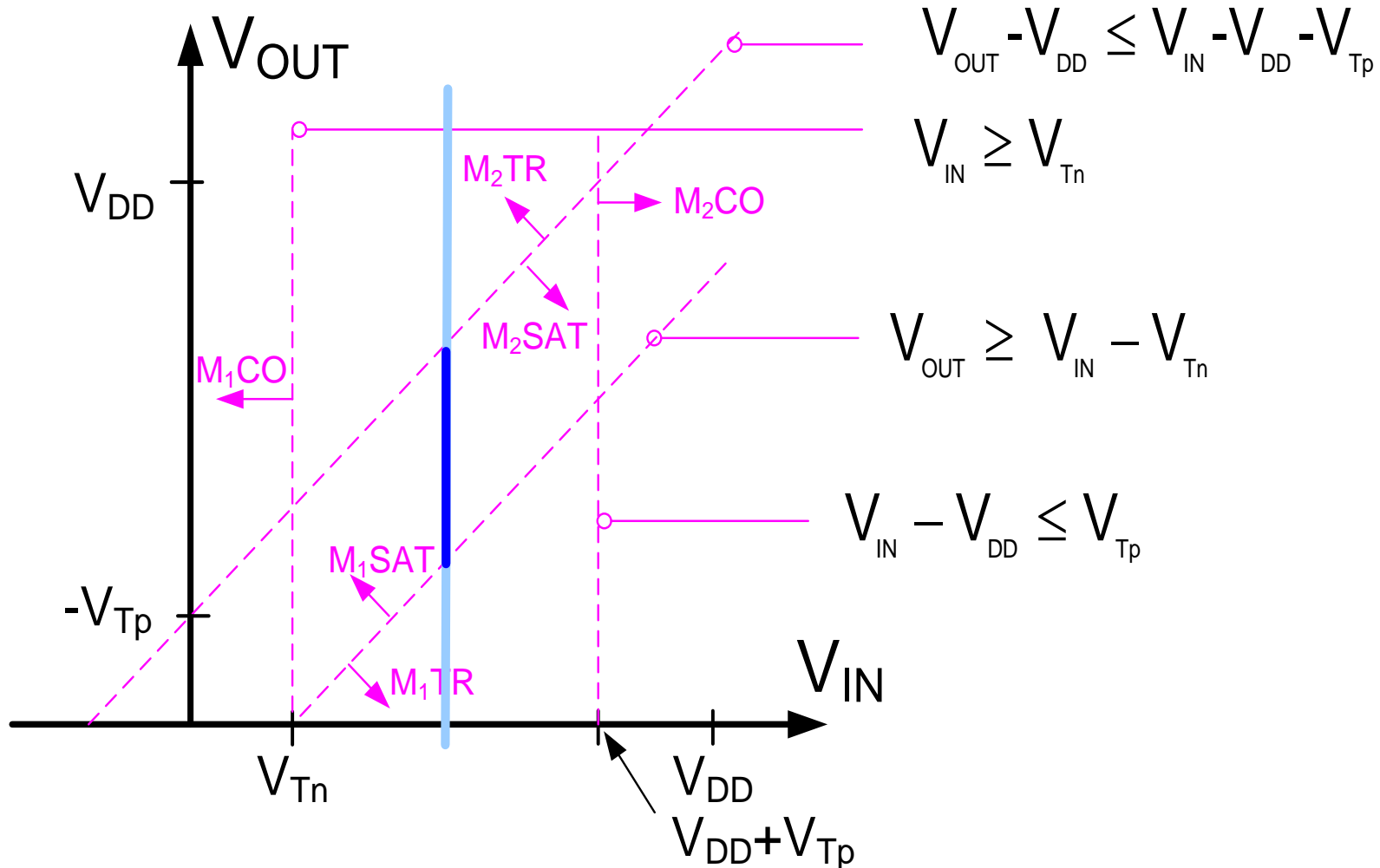
Case 3  $M_1$  sat,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

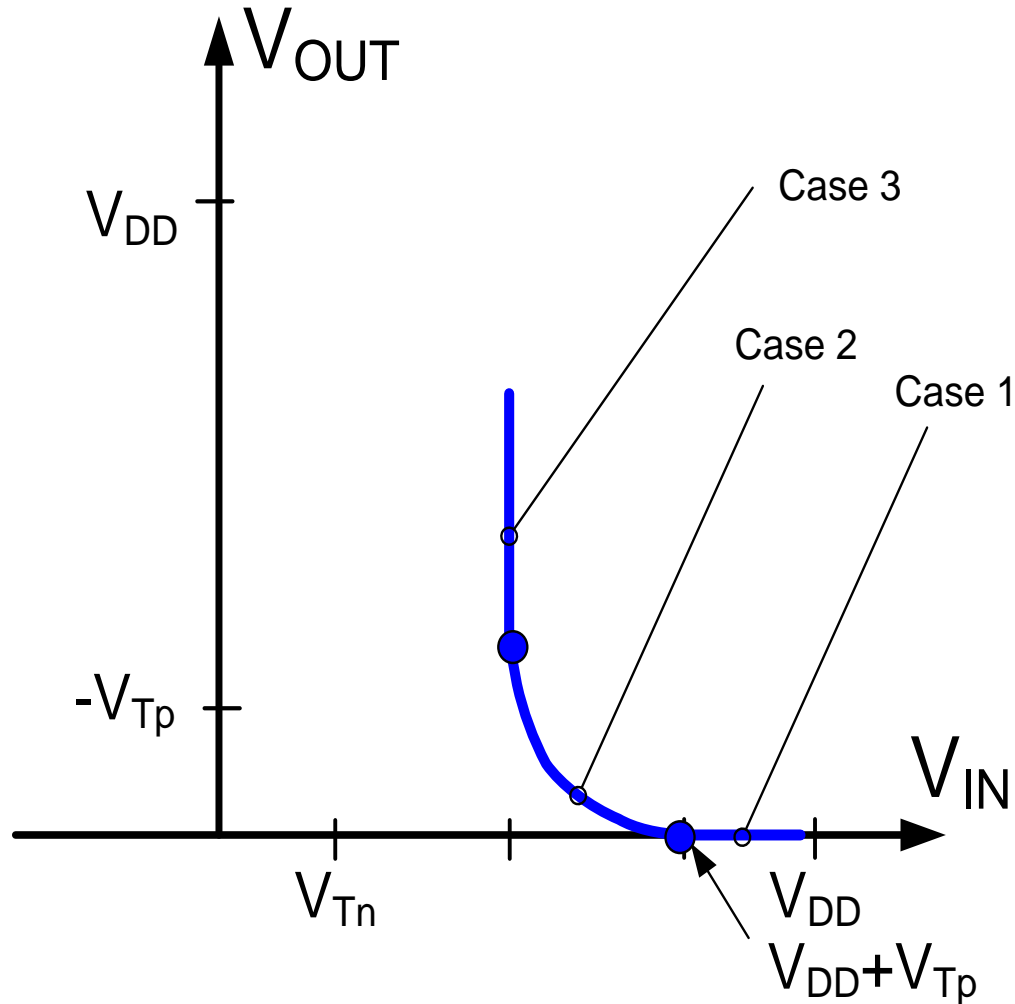
Case 3  $M_1$  sat,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

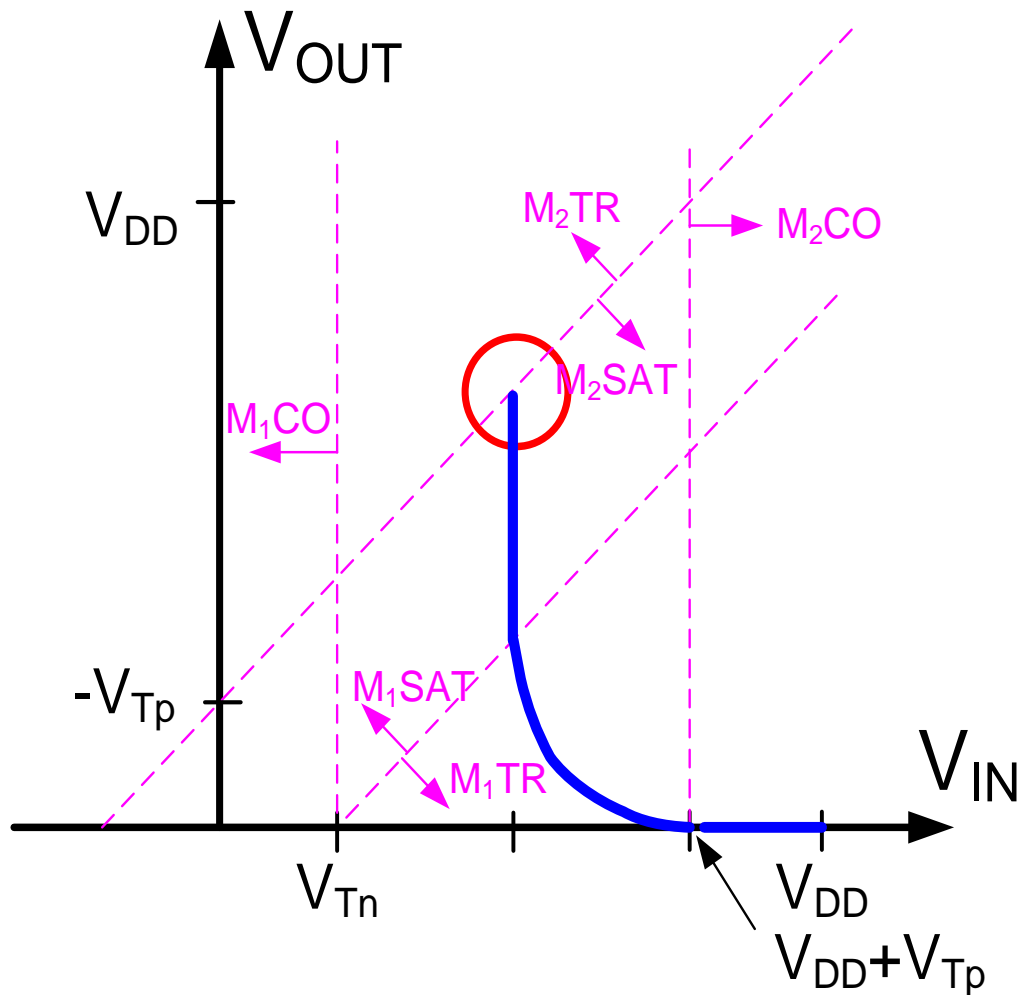
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode

$$I_{D1} = \frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

$$I_{D2} = -\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

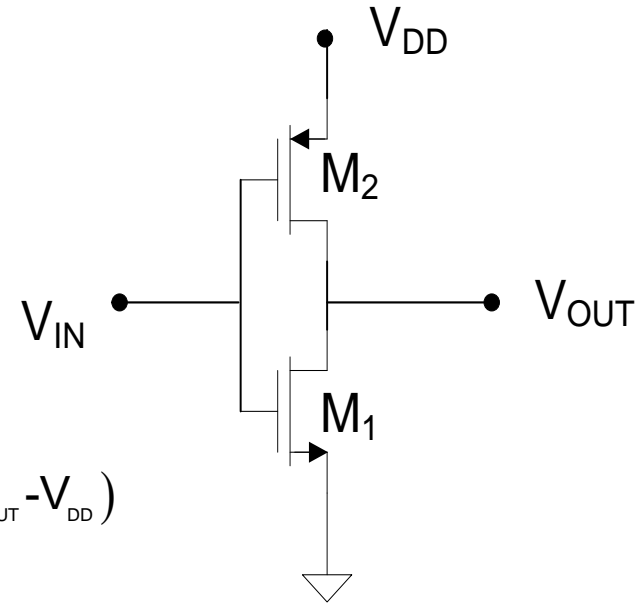
$$\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2 = \mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} \geq V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} > V_{GS2} - V_{Tp}$$

thus, valid for:

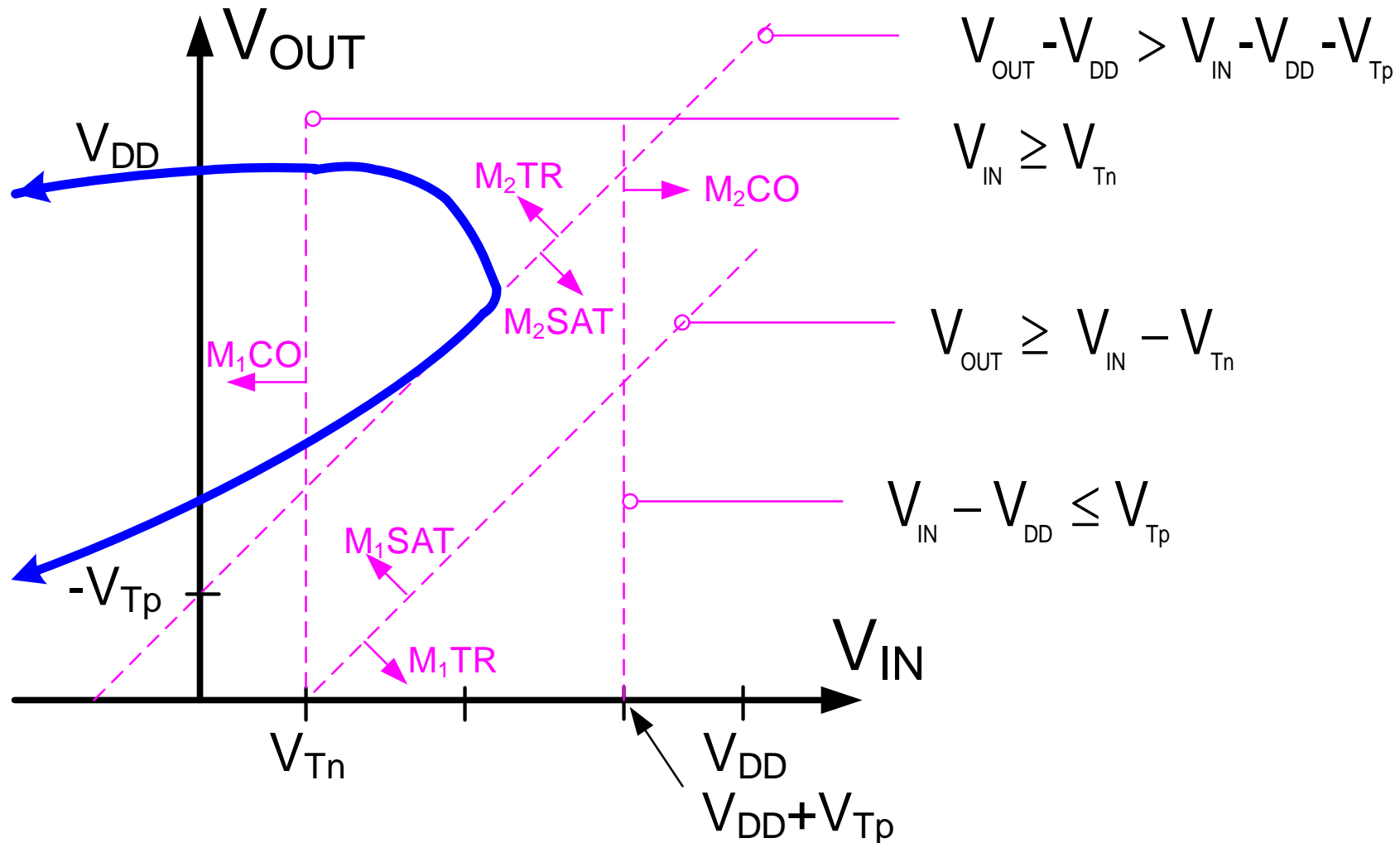
$$V_{IN} \geq V_{Tn} \quad V_{OUT} \geq V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode

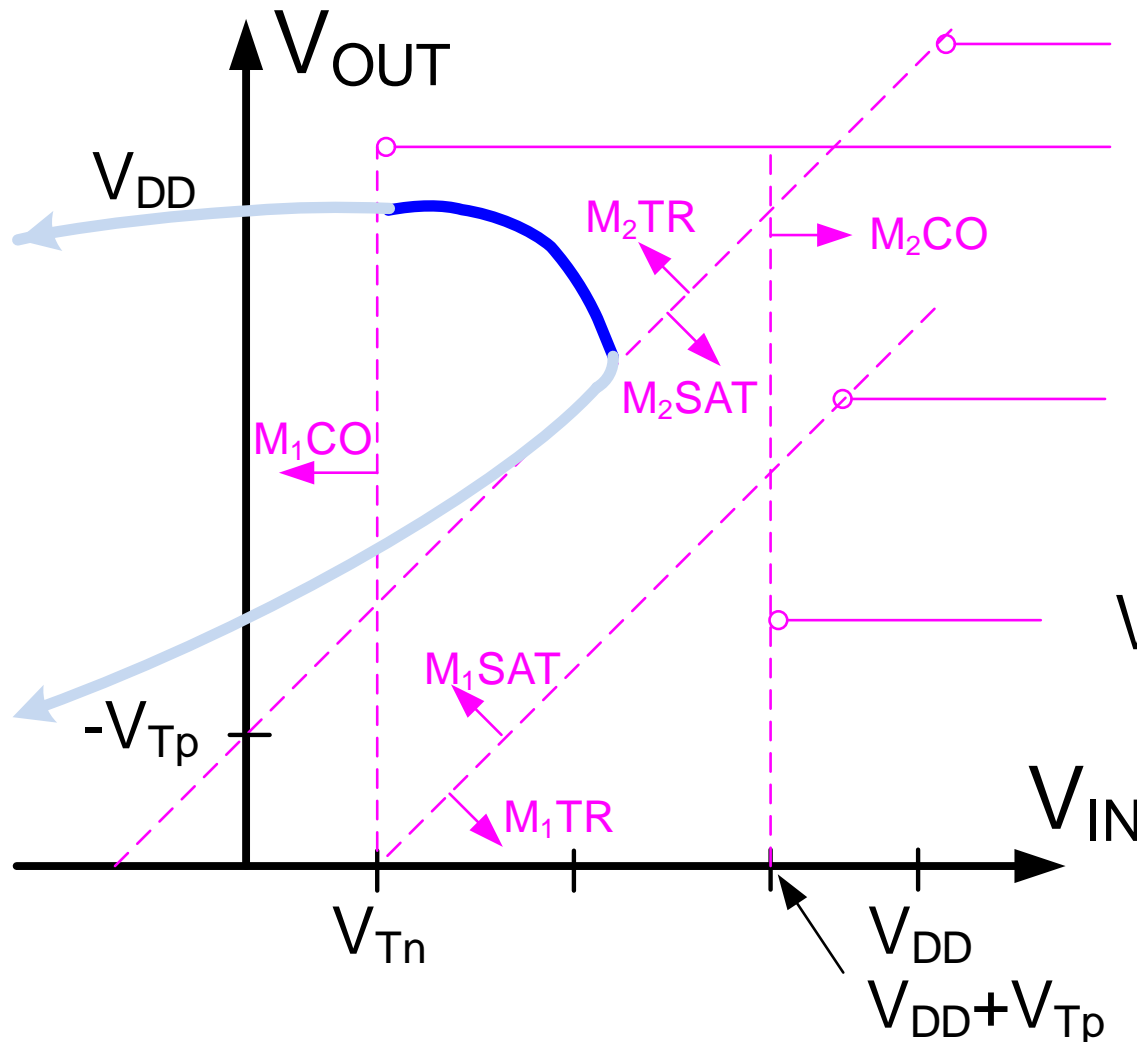




# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode



$$V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$

$$V_{IN} \geq V_{Tn}$$

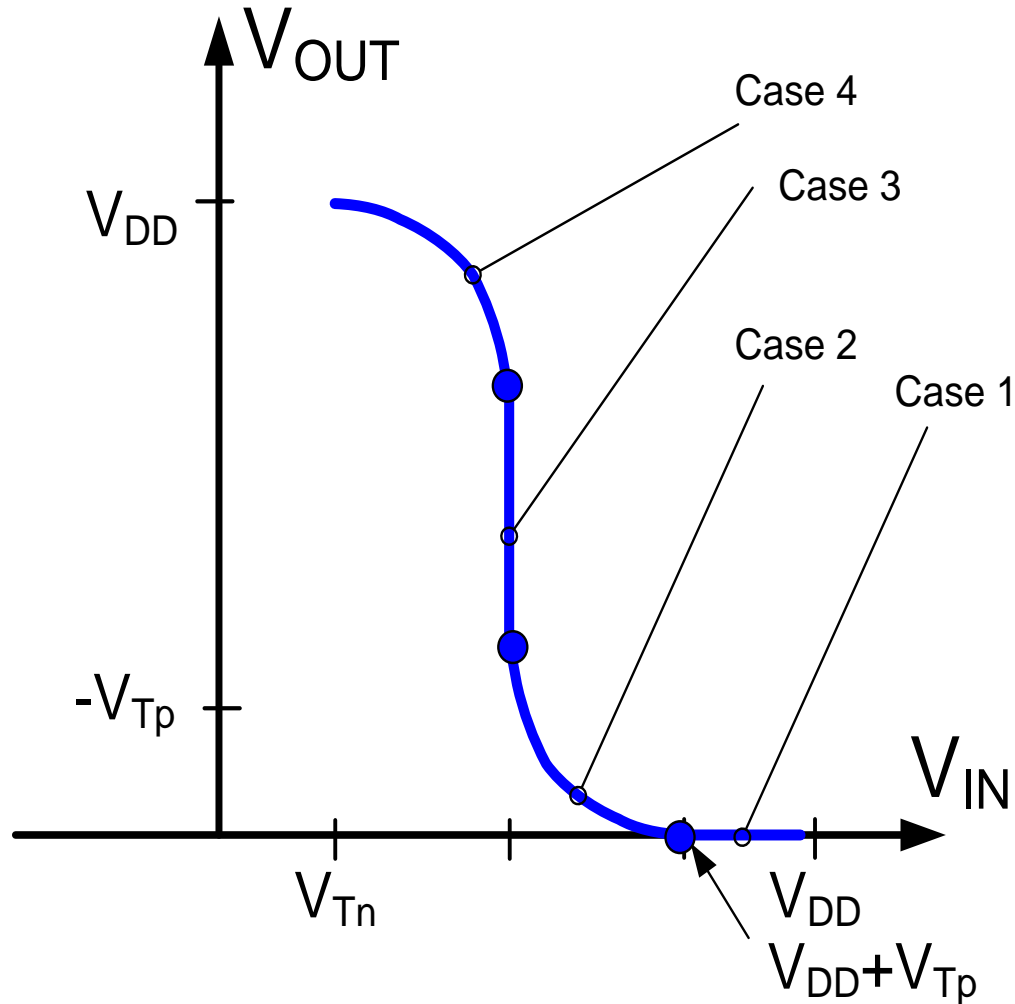
$$V_{OUT} \geq V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \leq V_{Tp}$$

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

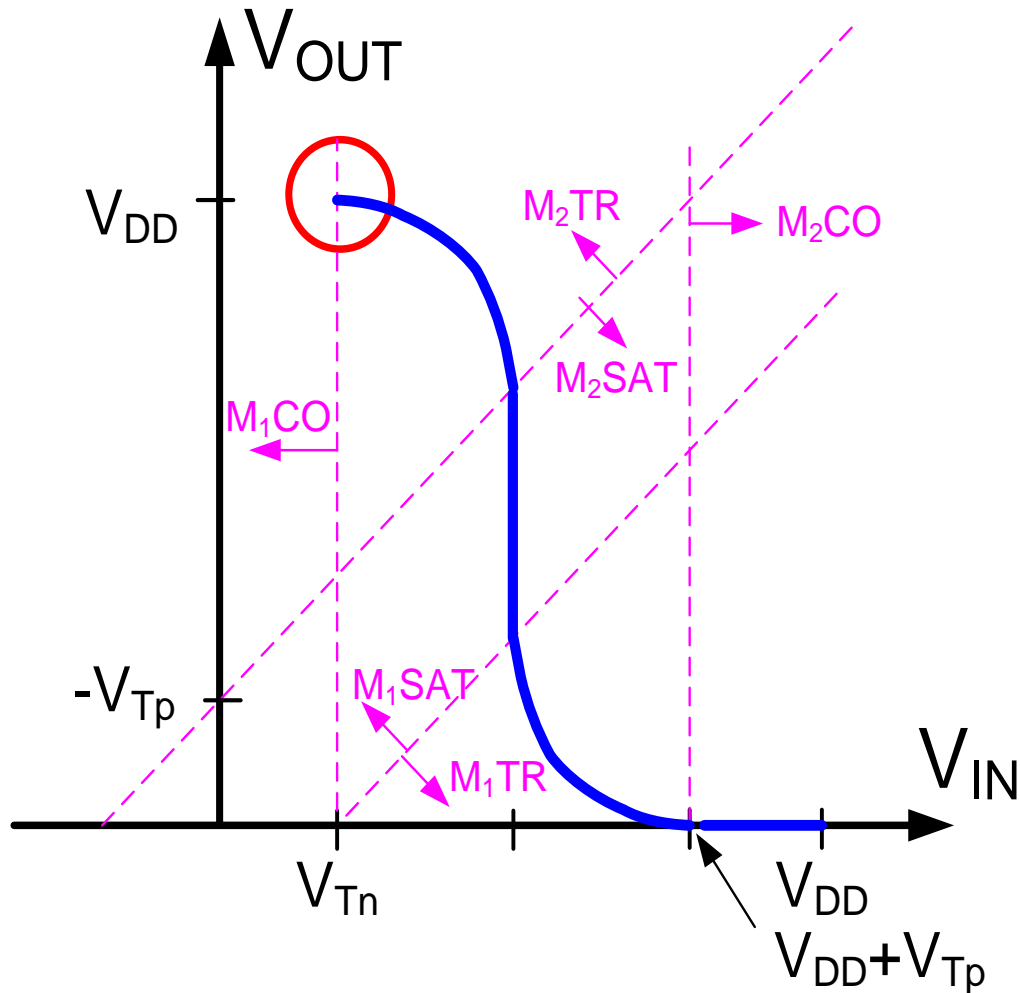
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  cutoff,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 5  $M_1$  cutoff,  $M_2$  triode

$$I_{D1} = 0$$

$$I_{D2} = -\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

$$\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD}) = 0$$

valid for:

$$V_{GS1} < V_{Tn}$$

$$V_{GS2} \leq V_{Tp}$$

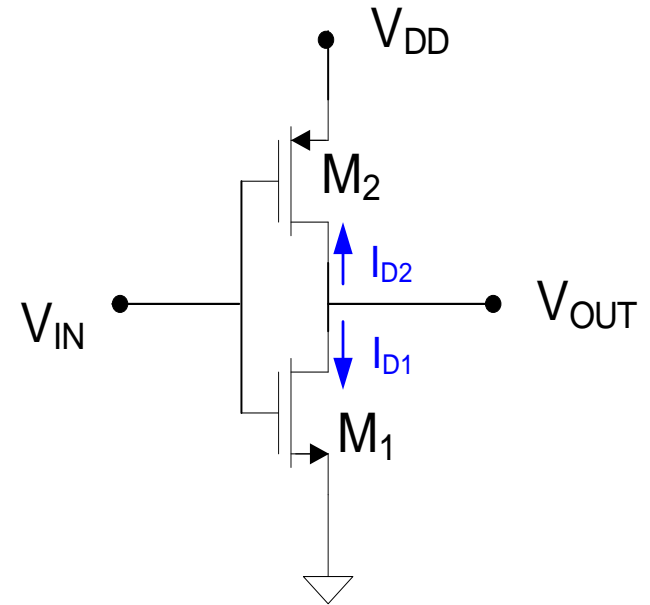
$$V_{DS2} > V_{GS2} - V_{T2}$$

thus, valid for:

$$V_{IN} < V_{Tn}$$

$$V_{IN} - V_{DD} \leq V_{Tp}$$

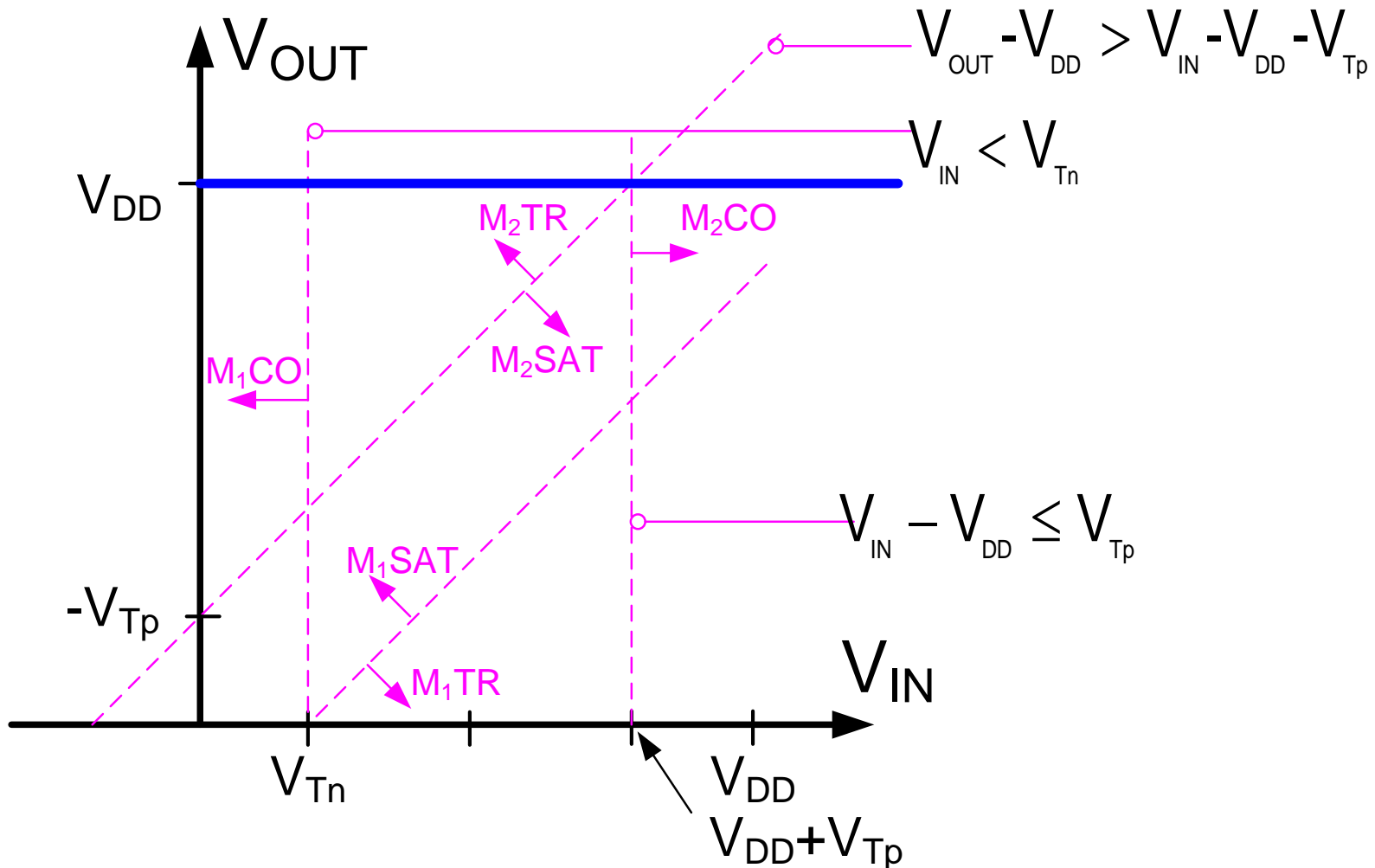
$$V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

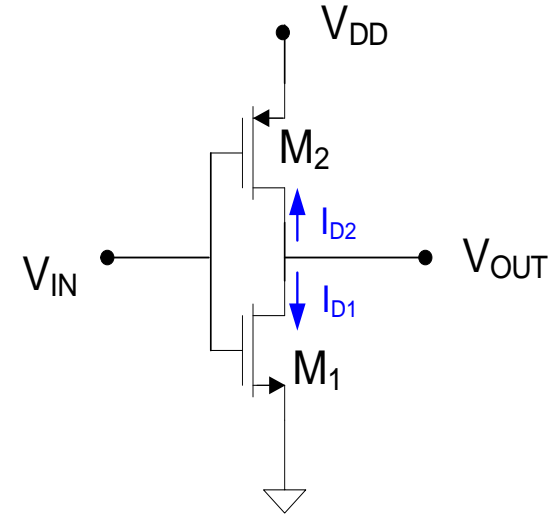
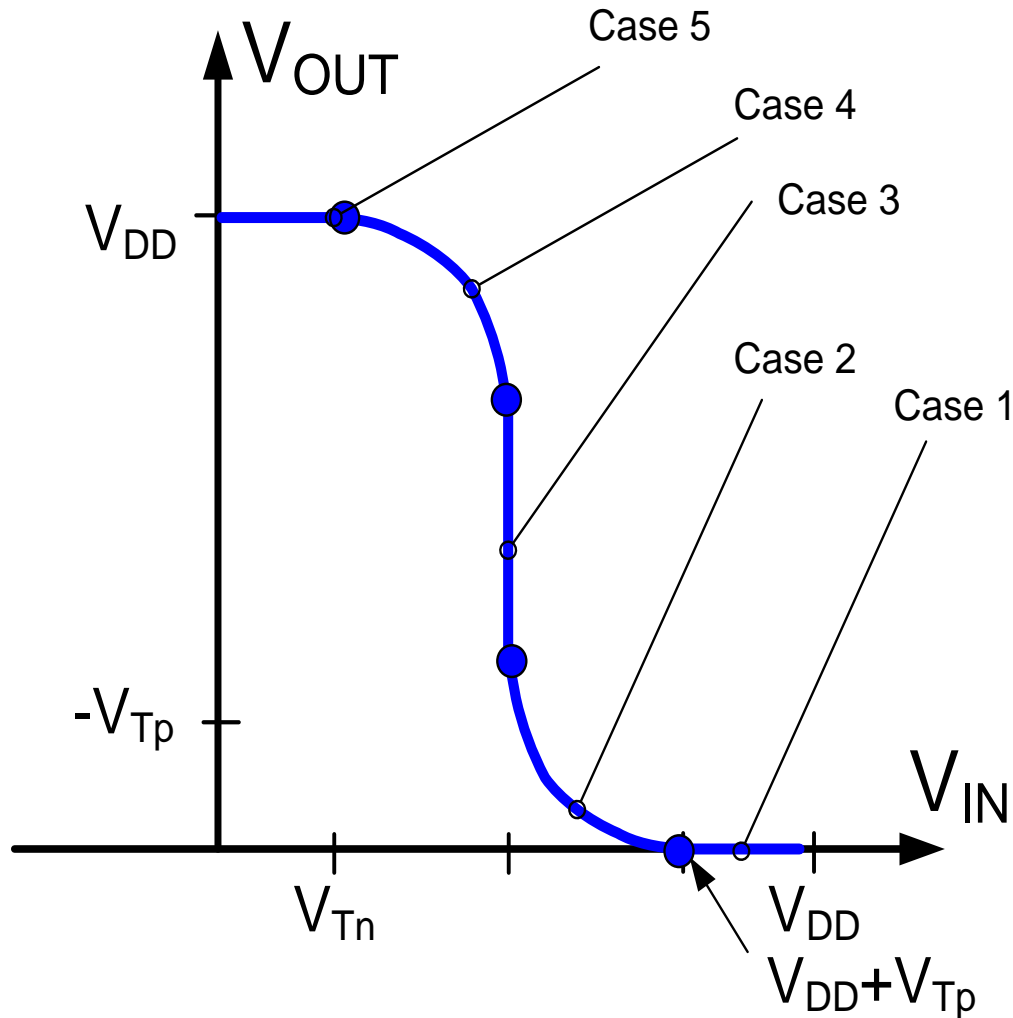
Case 5  $M_1$  cutoff,  $M_2$  triode





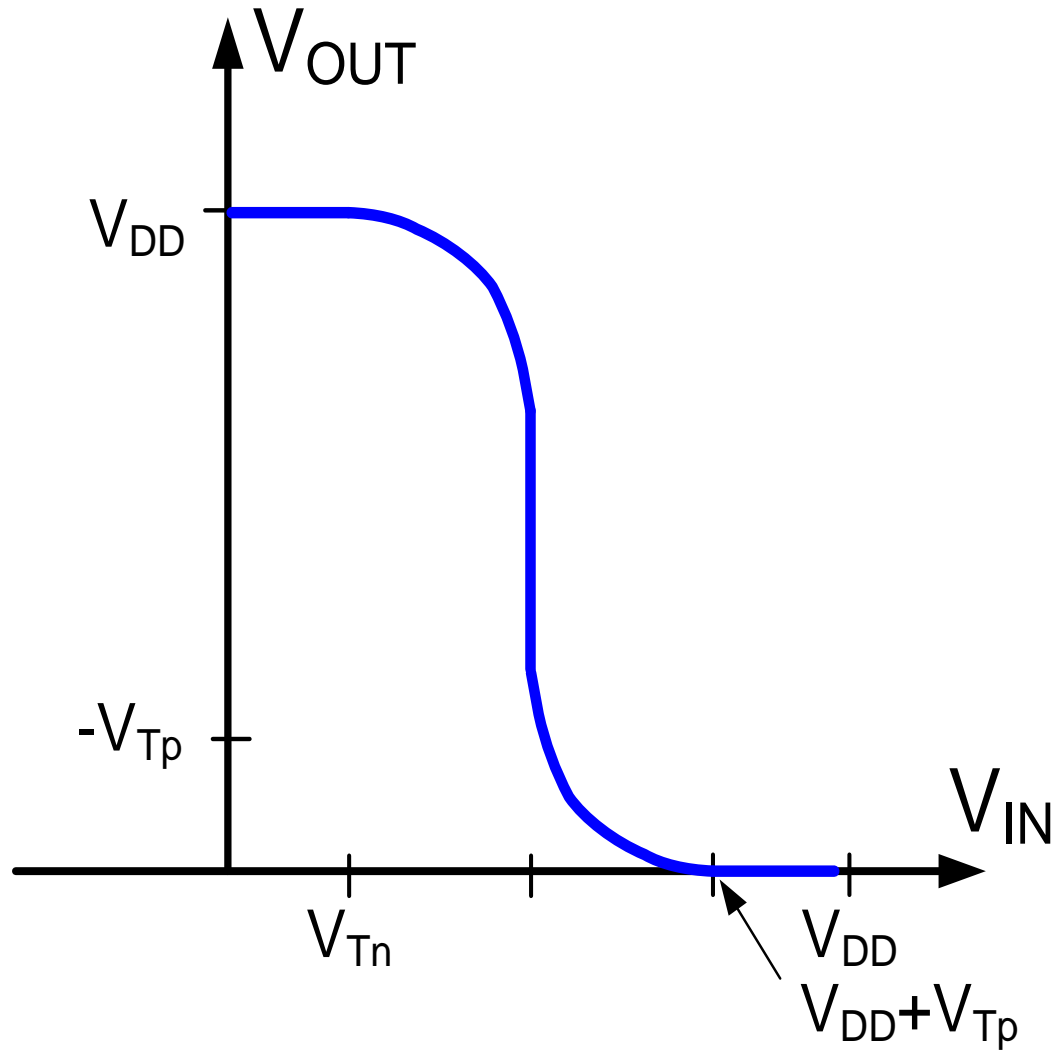
# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)



# Transfer characteristics of the static CMOS inverter

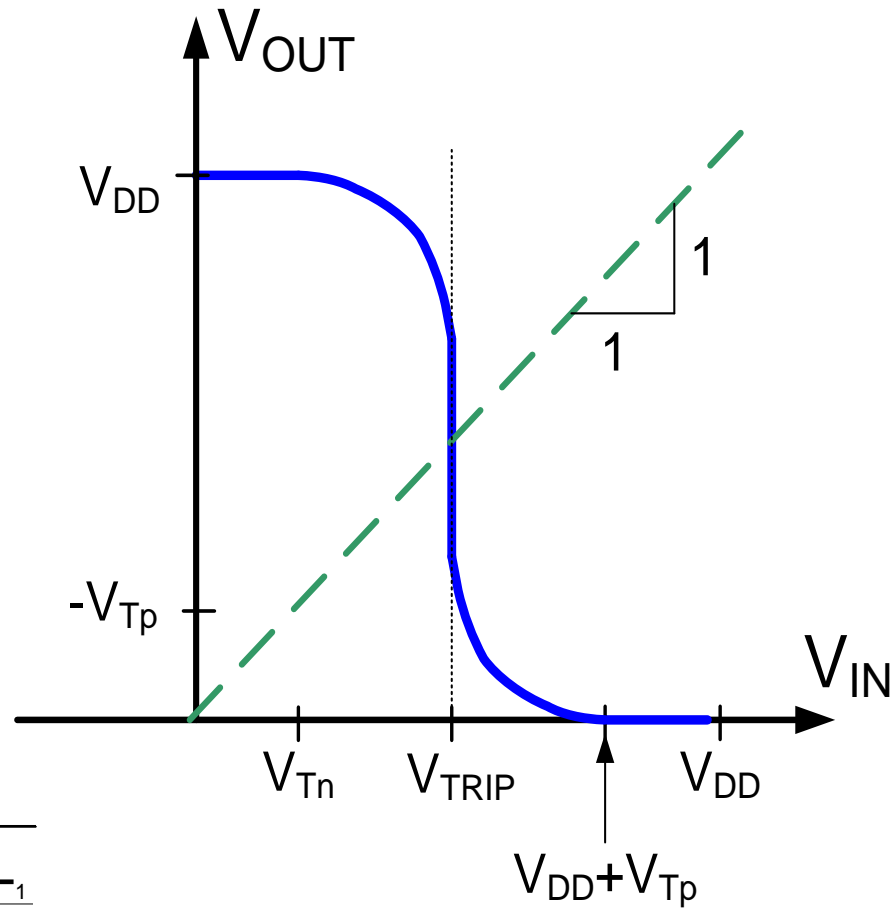
(Neglect  $\lambda$  effects)





# Transfer characteristics of the static CMOS inverter

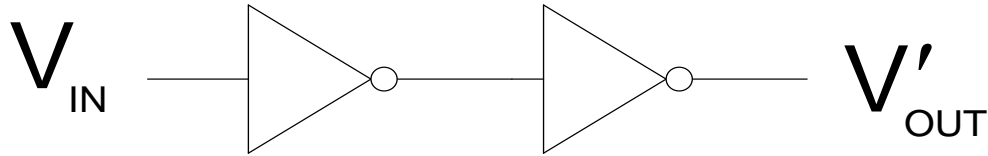
(Neglect  $\lambda$  effects)



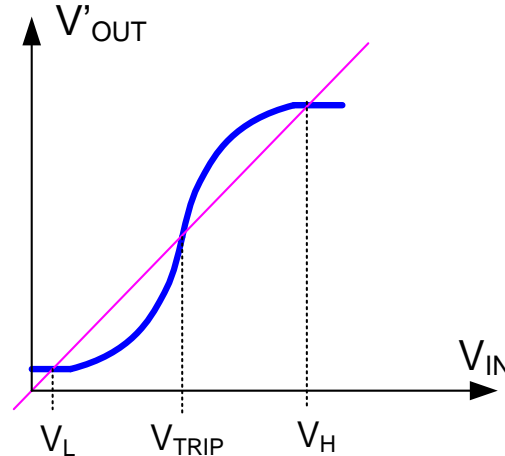
From Case 3 analysis:

$$V_{IN} = \frac{(V_{Tn}) + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}$$

# Inverter Transfer Characteristics of Inverter Pair

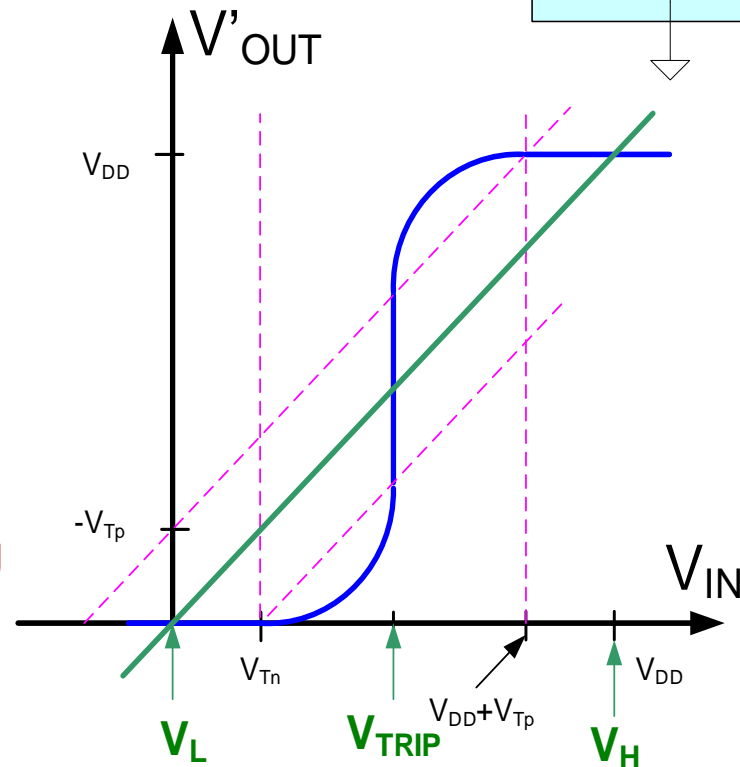
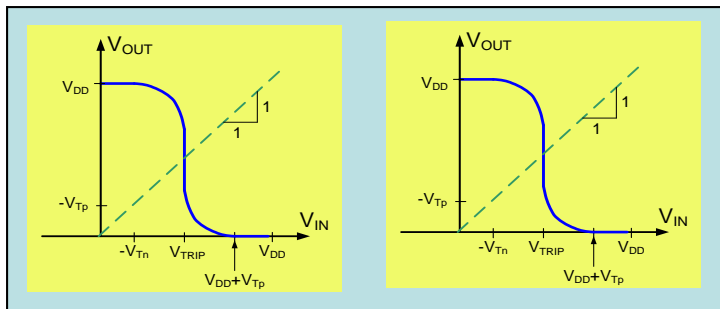
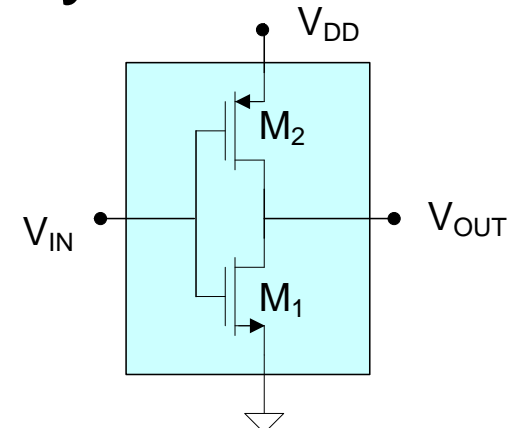
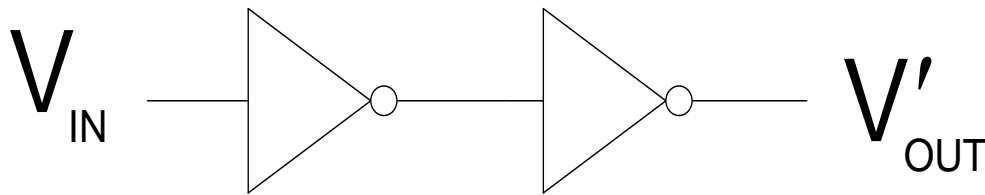


What are  $V_H$  and  $V_L$ ?



Find the points on the inverter pair transfer characteristics where  $V_{OUT}' = V_{IN}$  and the slope is less than 1

# Inverter Transfer Characteristics of Inverter Pair for THIS Logic Family



$$V_H = V_{DD} \text{ and } V_L = 0$$

Note this is independent of device sizing for THIS logic family !!

Designer can use sizing to achieve other desirable properties !!!



Stay Safe and Stay Healthy !

**End of Lecture 37**